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Cover Page Footnote

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Intriguing Interpretation of Dyads in Common-Practice Tonal Music

Yosef Goldenberg

The basic building blocks of common-practice tonal music are triads, and the music's basic texture comprises four parts. This is true already for most Baroque music that predates the conscious conceptualization of the triad, and remains true in chromatic harmony.[1] Nevertheless, doublings, rests, or consistently incomplete texture can give rise to dyads in tonal music, which normally imply complete harmonies. In such a situation, the actual auditory input does not include all that is conceived by listeners, consciously or not. Like a musical synecdoche, part of the chord stands for the entire chord and requires interpretation. The process of that interpretation, however, does not have simple analogues in literary or other arts.

Theoretical explanations and a cognitive understanding of how we know the complete chord usually rely on position finding of intervals in the key (Browne 1981, Egmond and Butler 1997). The current state of knowledge is at best incomplete, and the present study does not offer progress in approaching it. Nevertheless, in practice, most situations offer little controversy. For example, in the progression G/D/F/B–C/C/E/C, the C+E dyad stands for C major triad, with an implied G (V7–I progression). In a complete texture, the same dyad could be part of a deceptive

[1] Harrison (1994, esp. 57–64) proposes disassembly of chords into their components in chromatic music. Nevertheless, the involved sonorities remain of at least three distinct pitch classes.

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cadence (vi, with explicit A [$\hat{6}$]) or of other manners of "fuggir la cadenza."[2] However, when the dyad sounds unaccompanied, such options are unlikely to emerge as proposed alternatives.

While usually there is little room for interpretation in the completion of the dyad into a chord, the present study directs its attention to exceptional situations, where the meaning of a dyad changes. Evaluation of such situations should take into account two distinctions:

1. Sometimes the reinterpretation takes place when a dyad is restated (directly or in remote analogous locations, possibly in sequence or even in retrograde); alternatively, dyads may serve as actual pivots, implying different chords in the context of what precedes them and what comes after them (See below detailed discussion in relation to Example 4).
2. Often, dyads that change their meaning are located at formal junctures (e.g., retransitions). This is true both for pivot dyads and for dyads that change their meaning when repeated. Nevertheless, mid-phrase dyads may also be subject to reinterpretation.

The main organizing principle of the paper is the identity of the dyad (what interval it is and where it is in the chord and in the key): 1) Thirds that are members of the tonic triad, and their mediant or submediant potential; 2) Similar third-related interpretations of thirds in other locations in the key; 3) Thirds that are upper partials of seventh chords or further extended stacks of thirds, and other dissonant interpretations of thirds; 4) Thirds that are triad members but whose completion is dissonant; 5) Reinterpreted dyads that are not thirds. The final sections deviate from the criterion of the dyad's identity: 6) Enharmonically re-interpreted dyads; and, finally, 7) Dyads that do not imply complete harmonies.

[2] Exceptional deceptive cadences that incorporate both $\hat{1}$ and $\hat{3}$ are IV7 or (especially in minor) its chromatic inflection vii^{o}7/V. In a more radical procedure, $\hat{1}$ and $\hat{3}$ take part in an augmented triad, with explicit $\sharp\hat{5}$. See the first example in Appendix 1. For a general survey of deceptive cadences in the broad sense, see Neuwirth 2015.

I have chosen the harmonic identity of the dyad as the main organizing principle of the paper because I find it the most apt tool to explore the entire spectrum of possible dyads (in tonal music) whose completion into chords is not trivial. The main purpose of the systematic approach is to create a taxonomy of the existing examples found in the musical literature, but I shall also discuss the theoretical potential of dyads in the tonal system in general.

Along with illuminating the music examples, studying the spectrum of possible dyads is also an end in itself, and a theoretical innovation. Common wisdom is aware at best of the most ordinary configurations. The alternative criteria discussed above—how the dyad changes its meaning (as a pivot or when repeated) and the dyad's location in the form (at a juncture or mid-phrase)—cut across my seven categories that are based on a systematic study of the possible dyads. Appendix 1 enables an efficient tracing of the alternative distinctions. Directing attention to momentary sonorities should by no means cancel contextual listening nor should it focus on sonorities in isolation.[3] To the contrary, the interest in the dyads demonstrated in this study stems from the manner in which they lend themselves to more than one context.[4] A pure contextual listening, however, might dismiss altogether the non-structural surface dyads, and thus overlook the artistic devices that reinterpret them. I would endorse a multivalent listening strategy that involves awareness of surface elements such as emerging dyads (along with non-harmonic dissonances and non-structural motives), but explains them in context (or sometimes in more than one context). For example, when Beach (1967) studies the various functions of six-four chords in tonal music, he stresses the different contexts of cadential, passing, neighboring, or consonant six-fours, but their very grouping together under one study derives from their shared feature, namely the identity of their sonority as a simultaneity that does not depend on context. Many of the dyads we will encounter are not only far from being "salient features" of the music in a structural sense

[3] For an approach that highlights momentary sonorities in isolation, see Schoenberg ([1911] 1978, 324), discussing certain extreme dissonances in Bach and Mozart.

[4] Schachter (1969, 219) makes a similar case for a non-structural motive, praising its "changed contrapuntal relationship and the delicate shadings it produces."

(as Rothgeb [1997] advocates), they are also not always "immediately noticeable" (ibid, 181). Nevertheless, noticing them does enrich listening experience, and, in the final analysis, attending to them reinforces awareness to tonal context.

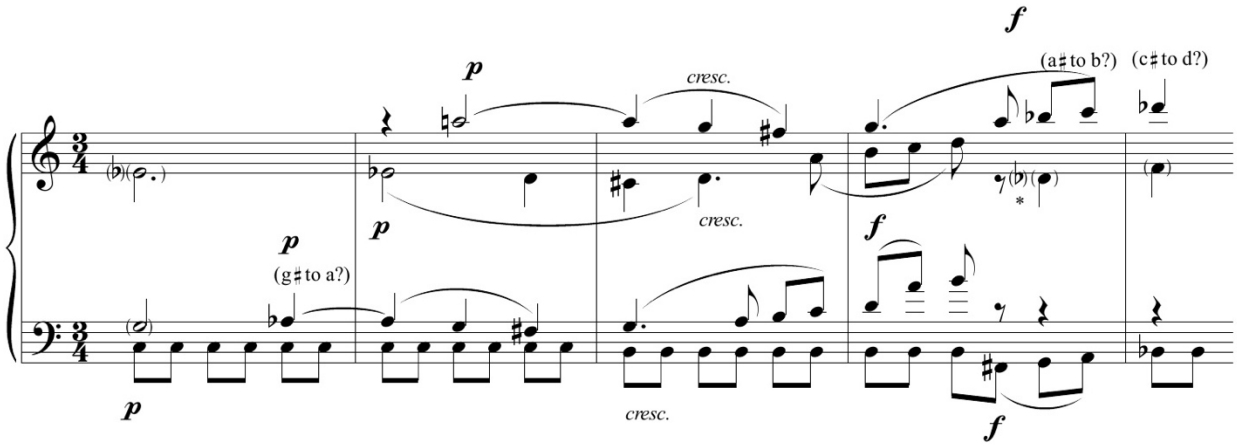
§1 LITERATURE SURVEY

Back in the third edition of *Versuch einer geordneten Theorie der Tonsetzkunst* (1830–32), Gottfried Weber did pioneering work on the *Mehrdeutigkeit* of dyads. An English translation of his work appeared as early as 1842, with the term *Mehrdeutigkeit* translated as "equivocalness." A more direct translation, "multiple meaning" comes from Janna Saslaw's "*Gottfried Weber and the Concept of* Mehrdeutigkeit" (Saslaw 1992), a comprehensive study of Weber's *Versuch*. Along with other types of multiple meaning, Weber ([1830–32] 1842, 228) presents, in an abstract manner, how dyads can be completed into various complete harmonies (triads or seventh chords). Reading that abstract presentation alone, one may get the impression that Weber is simply presenting the multiple meanings of dyads in various contexts, and is not claiming that this multiplicity creates ambiguity in a specific context.

In the appendix to the third volume (out of four in the third edition) of his treatise, Weber does apply his concept of multiple meaning to dyads in the context of a specific piece: the daring opening of Mozart's String Quartet K. 465 (Weber [1830–32], appendix to Vol. 3, 196–226, fully translated in Bent 1994, 161–87).[5] Example 1 follows Weber's analysis, indicating implied tones in parentheses.

[5] Earlier studies have already examined other aspects of this analysis: Saslaw 1992, 275–80; the foreword to the translation in Bent 1994, 157–160; Moreno 2003).

EXAMPLE 1. Mozart, String Quartet K. 465, i, mm. 1–5. Implied tones (in parentheses) and their potential accidentals (in separate parentheses) added after Weber [1830–32] in Bent 1994, 161–87. Asterisked flat is not after Weber.



Weber refers to three dyads in the passage (m. 1 beat 3, m. 4 beat 3, and m. 5 beat 1). For each of them, Weber makes the claim for enharmonic unrealized potential. Quite oddly, he finds tentative augmented intervals (one augmented fifth and two augmented seconds) when the actually realized meanings are ordinary intervals (one minor sixth and two minor thirds). The enharmonic transformation would be more convincing in the opposite direction (see below, Example 23).

A close scrutiny reveals that the dyads' ambiguity in Mozart's passage hardly depends on the incomplete texture. Weber focuses on the ambiguous (enharmonic) potential of explicit members of the dyads, not on the uncertain identity of the implied tones, or, put another way, the uncertain completion of the dyads into complete chords. Uncertain completion is indeed the case in m. 1, even before a dyad even sounds. Weber notes: "Right at the outset, the bass note C sounds in isolation [*ganz allein*]. . . . The ear is inclined initially to hear [either] C major [with implied E] or C minor [with implied E♭]." At the last beat of the measure, "the note A♭ enters against this C. This leaves the ear with a new element of uncertainty: is this latter note to be heard as G♯ or A♭?." The uncertain completion of the chord is still implicit in that a reading as G♯ that resolves to A

could only work with implied E, not with E♭. Concerning the dyad in m. 4, the situation is different. Weber only considers options with the implied D; the alternative implied D♭ is my own suggestion. The ambiguous aspect that Weber does discuss concerning that dyad [*Zusammenklang*] is the potential enharmony of the upper voice. But this ambiguity would work the same had the sonority been played complete (with explicit D). The same is true for the dyad in m. 5.

As a reaction to the position stated by Weber (but without direct reference to Weber), V. Kofi Agawu (1994, 88) claims: "once the enabling constructs of music theory are brought into play, equivocation disappears." Agawu's position is certainly correct in respect to the abstract potential of dyads. Nevertheless, that position is possibly an overstatement: in exceptional cases such as those demonstrated in this paper, the equivocation created by dyads remains in effect. Weber himself was probably aware that the ambiguous potential is only realized in specific circumstances, as he indicated (in different context, not concerning dyads) "really or properly equivocal[ness]" [*eignetliche oder wirkliche Mehrdeutigkeit*] (Weber [1832] 1842, 369 [§218] = 1832, vol. 2, 156).

The dyads Weber shows in K. 465 could count as a weak case of "really equivocal" dyads. The alternative, augmented, meaning of the sounded intervals becomes possible in real-time listening only, but is removed in retrospect. Moreover, even as a momentary unrealized potential interpretation, the alternative meaning is not strongly preferable to the reading that is eventually taken. In a modern account of Weber's ideas, Damschroder (2008, 18) demonstrates a dyad (discussed below as Example 5) that is more strongly ambiguous than any dyad Weber discusses himself: it serves as a pivot. Pivot dyads in particular pose an exception to Agawu's position, since they deliberately build on the incompleteness of the data in a manner that defies a single meaning. In such pivots the implied complete chord in relation to the preceding material is usually clear (unlike in Weber's own demonstration), and the different implied complete chord in relation to the forthcoming material is clear as well.

Current analytical literature hardly engages dyads. In a single study that focuses on dyads in tonal music, Michael Baker (2009, see Example 3 below) discusses certain cases of dyads that change their meaning when repeated as a "curious type of tonal pun." Baker brings together many interrelated examples (see also Appendix 1). Not all of his examples use dyads, but those that do are very similar to one another, using the most common type of dyad usage.

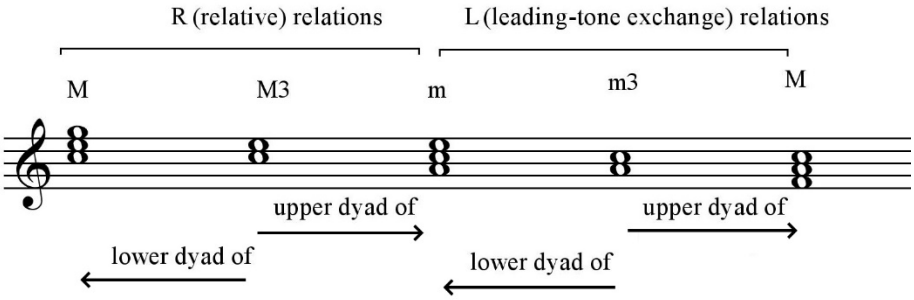
Further analytical references to dyads that change their meaning appear ad-hoc in the course of discussion of other issues. I will refer to these in footnotes (Webster in note 7; Hepokoski and Darcy in note 11). Thus, theorists that encounter actual examples of dyads whose completion is unclear or subject to change do indeed grasp the situation in much the same way proposed here. Awareness of the existence of such dyads is thus not new. However, there is a very large theoretical lacuna that the present paper aims to fill: no earlier study explores systematically the circumstances that may give rise to changes in the tonal interpretation of dyads. In the spirit of Weber's title, this study offers for the first time a true attempt at a systematically arranged theory of dyads.

§2 THE BASIC AMBIGUOUS DYADS: $\hat{3}$+$\hat{5}$ AS TONIC OR MEDIANT AND $\hat{1}$+$\hat{3}$ AS TONIC OR SUBMEDIANT

The basic type of dyads that lend themselves to more than one completion is consonant thirds. These thirds can serve as either an upper or a lower pair of members of a triad. This is by far the most common situation of an ambiguous dyad. As is now common knowledge, also thanks to neo-Riemannian theory (see Example 1), a major third can serve as the lower dyad of a major triad or as the upper dyad of its relative minor triad (whose root lies a third lower than that of the major triad). A minor third, by contrast, can serve as the lower dyad of a minor triad or as the upper dyad of a major triad in leading-tone exchange relations to it. In that case, it is the major triad whose root lies a third below that of the minor triad. Such dyads are an important component in tonal pairing of third-related key-complexes (Bailey 1985, 121–22). While single tones serve well

chromatic third relations (Rothstein 2008), the ordinary implication of shared thirds is limited to diatonic third relations.

EXAMPLE 2. Available contexts for major and minor thirds in consonant triads, with neo-Riemannian labels



The most common position of these thirds is as members of the tonic ($\hat{1}$+$\hat{3}$ or $\hat{3}$+$\hat{5}$). It is the incomplete texture that enables the aforementioned "tonal pun" Baker (2009) discusses. Baker has shown multiple related examples of this procedure, most of them based on $\hat{3}$+$\hat{5}$ in minor as either i or III (more usually with the completion into a minor triad first).[6]

Example 3 shows a typical example from Baker (newly engraved with further annotations). The second section of a binary bourrée in C minor opens with an unaccompanied E♭ that moves to G via a passing tone. When it is first heard after a cadence in the relative major, it continues that harmony (E♭ major). When, however, the second section is repeated after a cadence in the tonic, the same E♭ and G may sound as the third and fifth of C minor. These E♭ and G form an actual dyad (even though it is not simultaneous), but the third member of the triad has to be inferred from the context: in the first instance the implied tone is B♭, while on the repeat the implied tone might

[6] My Example 3 follows Baker's examples 13–14. See Appendix 1 for Baker's similar examples. In some other examples by Baker, the reinterpretation is not applied to dyads: in his example 3 the subject of reinterpretation is a single tone, and in his examples 12 and 22 the very identity of the involved simultaneities is modified.

be C, the continued preceding stable tone.[7] Admittedly, the clear articulation of a repeated beginning after a clear cadence in the tonic should cause experienced listeners to recognize the repeat of the second section, and thus encourage them to hear III again in the renewed beginning. However, such renewed beginnings are also a familiar site of reinterpretation, as when an opening strong tonic becomes an apparent tonic at the beginning of a recapitulation or a repeated rondo refrain.[8] Most often, there is no alternative to the reinterpretation, as when—to take one example among many—the recapitulation of Beethoven's Symphony No. 6, i, quotes the opening tonic material at pitch but, arriving after IV, it becomes an apparent tonic passing between IV and V (Rothgeb 1990, 11–12, after Schenker, not shown).[9] The excerpt in Example 3 is more intricate, and the reinterpretation remains optional, since the progression i–III at the point of repeat is more acceptable than a reading of a genuine tonic between IV and V (as would be the alternative in Beethoven's Symphony No. 6).

A factor that does strengthen the sense of reinterpretation in Example 3 is that the second section's opening moves immediately to the dominant of E♭, using A♮ without an earlier restatement of A♭. Arriving from C minor, the progression c-F-B♭ can smoothly replace the original E♭-F-B♭.

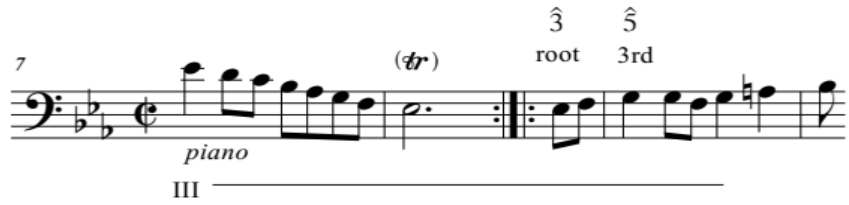
[7] When the dyad $\hat{3}$+$\hat{5}$ opens a piece in minor, listeners may be deceived into thinking that the piece is in the relative major. This happens in the much-discussed opening of Haydn, String Quartet Op. 33, No. 1, i. See discussion in Webster (2004, 128)

[8] For a general study of that phenomenon, see Wagner 1986, esp. 136–39.

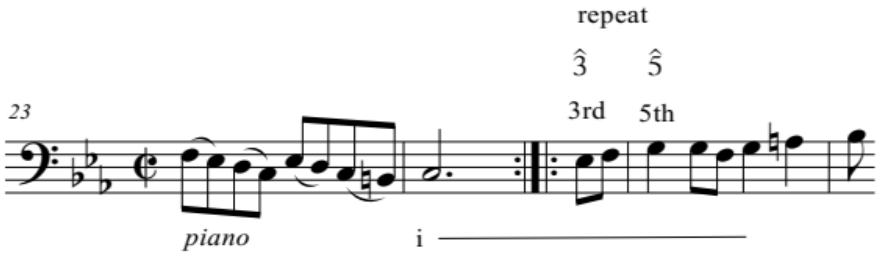
[9] As in the case that note 4 above discusses, an analysis that would exclude the apparent tonic altogether would miss its being a reinterpretation of the true tonic from the exposition (and the beginning of the thematic rotation of the recapitulation).

EXAMPLE 3. Bach, Cello Suite No. 3, BWV 1009, Bourrée No. 2. $\hat{3}$+$\hat{5}$ as root and third of III reinterpreted as third and root of i (after Baker 2009, 14).

(a) end of part 1 and beginning of part 2

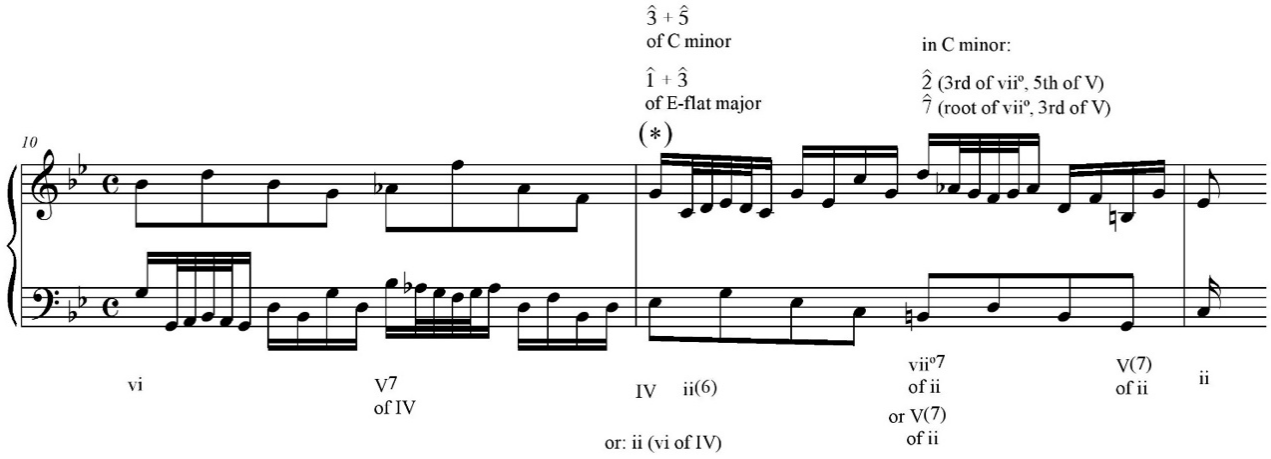


(b) end of part 2 and beginning of repeated part 2



In Example 4, the first sixteenth note of m. 11 is again a major third that is either the lower dyad of a major triad or the upper dyad of a minor triad, even with the same pitch classes E♭ and G. Here and in other examples, asterisks indicate the main dyads under discussion.

EXAMPLE 4. Bach, Invention in B♭ major, BWV 785. $\hat{1}$+$\hat{3}$ as resolution or incomplete deceptive cadence, depending on harmonic rhythm.



The beginning of m. 11 in Example 4 also differs from Example 3 in several respects, summarized in table 1.

TABLE 1. Comparison of Example 3 and Example 4.

	Example 3 after double bar	Example 4, m.11 first dyad
quality of interval	major third	major third
key relations	relative	relative
quality of tonic triad	minor	major
quality of relative triad	major	minor
function of relative triad	mediant	submediant
structural level affected	large-scale	surface detail
manner of change	under repeat	pivot

Unlike Example 3, in Example 4 (first dyad in m. 11) the (momentary) tonic is major and its relative minor is its submediant,[10] the dyad is a tonicized detail on the surface level with no effect on larger levels, and most significantly, it is not when the dyad recurs that its meaning changes (as in Example 3). Rather, in Example 4, a single statement of a dyad has different implications in the context of the way it is approached, and the way the music proceeds from it: it is a pivot.

In real-time listening, the pivot third in Example 4 resolves the preceding secondary dominant seventh (arpeggiated) chord, and there is no reason to assume any completion other than an E♭ major triad (IV). However, on the very next sixteenth note, the tone C belongs to ii (vi of

[10] For a case where a dyad serves as vi that becomes I in major at a structural point, see the retransition (m. 395) in the overture of Mendelssohn's *Midsummer Night's Dream*. Rosen (1980/88, 274) observes: "the E and G♯ [$\hat{1}$ and $\hat{3}$] of the flutes are placed over the C♯ of the strings, who then retire. . . . The main theme returns before the tonic harmony, and it is the theme itself which forms the retransition [directly from vi]." Rapoport (2012, 85–86) provides further analysis.

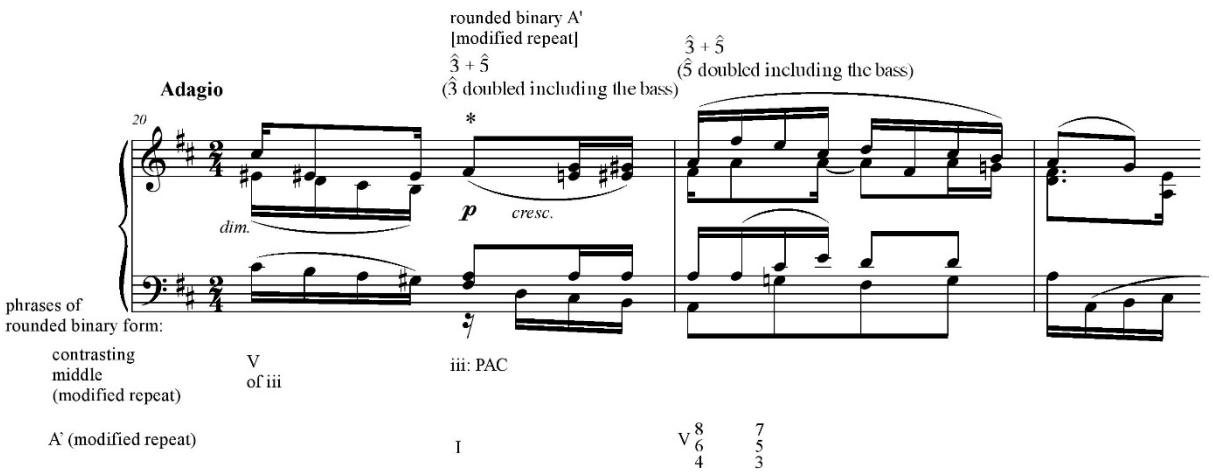
IV). Potentially, one could read a very quick 5–6 change, but such a reading would not fit the regular and much slower harmonic rhythm (two chords per measure). As the bass also descends to C within the time-span of a single harmony, the continuation of the phrase makes it more logical to hear a C minor harmony—ii—throughout the first half of m. 11, initiated already at the statement of the dyad. The experience of listening to the whole passage as it is indeed raises conflicting clues—as if the dyad stands for both triads.

Before leaving Example 4, let us look briefly at the first dyad in the second half of m. 11, B+D. Is the harmony a local V already since the start of the beat, or only when the root of that V arrives at the end of the measure? This dilemma might look similar to that in the first half of the measure, but the difference between the variants of reading B+D is mild. The progression vii^{o}7–V7 is almost a circular prolongation of V7, since all the tones of the vii^{o} triad are contained in V7.

Pivot dyads differ from ordinary modulatory pivot chords. In pivot chords, the identity of the tones in the pivot is the same backward and forward, and only the chord's role in the source key and the target key is subject to change. Indeed, when pivot chords are enharmonic, the spelling of some tones in the pivot changes and thus the meaning of the chord itself undergoes re-interpretation. Yet, pivot dyads involve a different kind of complication. They are perceived as incomplete chords (usually triads), but the identity of the additional, implied, tone (and thus of the complete chord) changes.

A similar situation exists in Example 5. Here, the dyad is a minor third moving in an overall major key (D major) from the tonicized diatonic mediant (F♯ minor) back to the overall tonic. Now, the connected keys are not relative but involve neo-Riemannian L (leading-tone exchange) relations.

EXAMPLE 5. Mendelssohn, Song without Words, Op. 102, No. 2, mm. 20–22. $\hat{1}+\hat{3}$ as cadence on iii and recapitulation on I (after Damschroder 2008, 18).



Following the theoretical conceptions by Weber discussed above, Damschroder (2008, 18) asks: "Do the pitches F♯ and A at m. 20, beat 2, represent tonic in F♯ minor, tonic in D, or both?" Example 5 takes place at a point of formal juncture. Within the modified repeat of the latter half of a rounded binary form, the contrasting middle ends with a perfect authentic cadence in iii, which elides with the beginning of the A' phrase in the tonic. While the bass of the tonic enters on the second sixteenth note of the second beat, the harmonic rhythm determines that the tonic already arrives on the beat. It is the incomplete texture that enables the elision of iii and I.

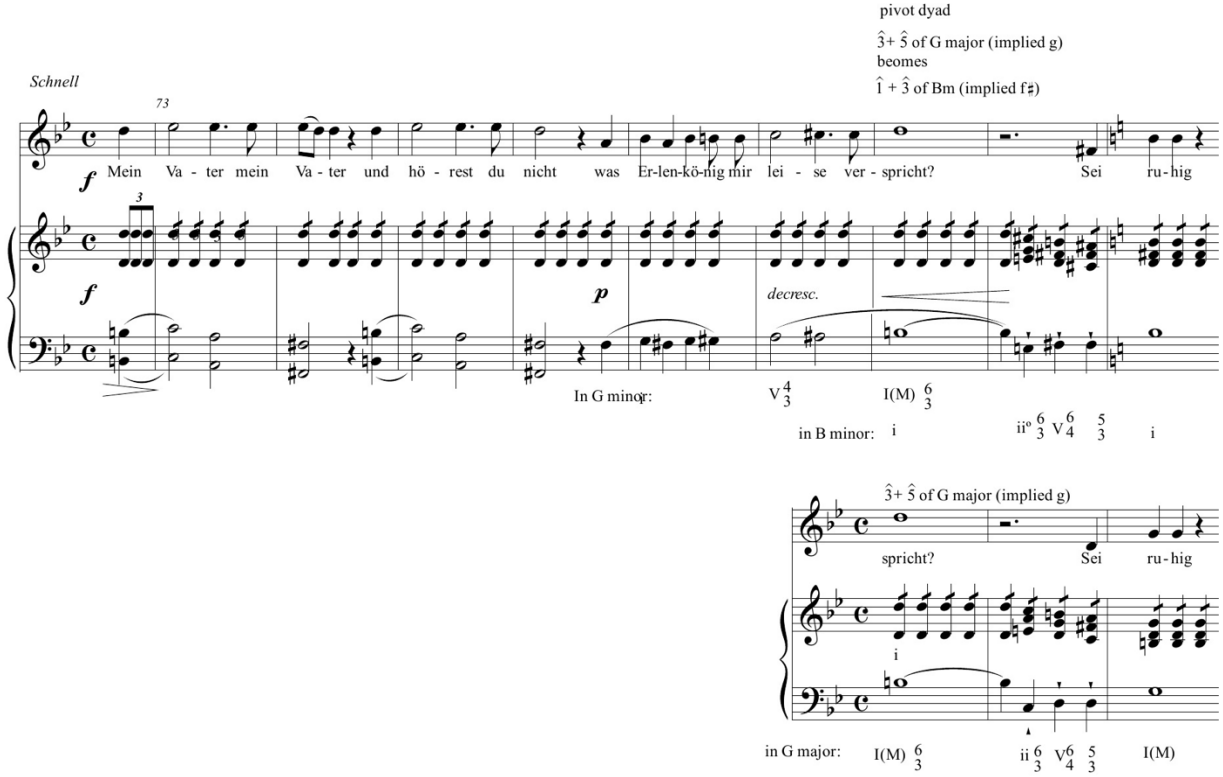
As with reinterpreted repeats, pivot dyads also are especially likely to occur at boundary points between formal sections, although—recalling Example 4—this need not be the case.[11]

Example 6 also presents a pivot dyad. Like Examples 3 and 5, Example 6 also comes from a point of formal boundary. However, the character of the juncture is different in each case: in

[11] In Example 5, a bass tone that is added to a dyad changes the harmony into a triad whose root lies a third lower. This is the complementary procedure to that in the Mendelssohn example cited in note 10, where the bass tone is removed, changing the harmony into a implied triad whose root lies a third higher. For discussion of another pivot dyad at a retransition, see Hepokoski and Darcy (2006, 279) on Schubert, Symphony No. 9, i, 591–98.

Example 3 the borders between sections are clear, and the reinterpreted dyad is the beginning of a section; in Example 5 the dyad serves as both ending and new beginning in a quite paradoxical manner; in Example 6, from Schubert's famous song *Erlkönig*, the pivot dyad takes place within a transition passage between disjunct units (to use a deliberately neutral term). In the fourth stanza of the text (mm. 73–86), as the psychological separation between the child and the father becomes acute, the music breaks the stanza into two pairs of lines separated by a modulation in chromatic relations. It is at the point of modulation that the pivot dyad occurs.

EXAMPLE 6. Schubert/Goethe, *Erlkönig*, mm. 73–81. Pivot dyad in a modulation: $\hat{1}+\hat{3}$ (I6/3) of G major = $\hat{3}+\hat{5}$ (i5/3) of B minor: Annotated score with hypothetical non-modulatory recomposition.



The child's part is in G minor, ending on the tone D with B♮ in the accompaniment. Arriving from G minor via a chromatic series of parallel tenths, the dyad stands for a G major triad. The unaccompanied dyad sustains over a complete measure, leaving time to imagine an ending before the actual continuation takes place. The precise context of the G major triad offers

two possibilities. According to one listening strategy, the tonic triad at the downbeat of m. 77 properly resolves the preceding dominant, after which the imaginary continuo remains G, and the implied G major triad in m. 80 is in root position. Alternatively, the dominant of G minor retains throughout the passage. In that case, the imaginary continuo in m. 79 is still D, and the implied G major triad is a 6/4 chord. In both variants, the tone G is implied at the moment of the D+B♮ dyad.

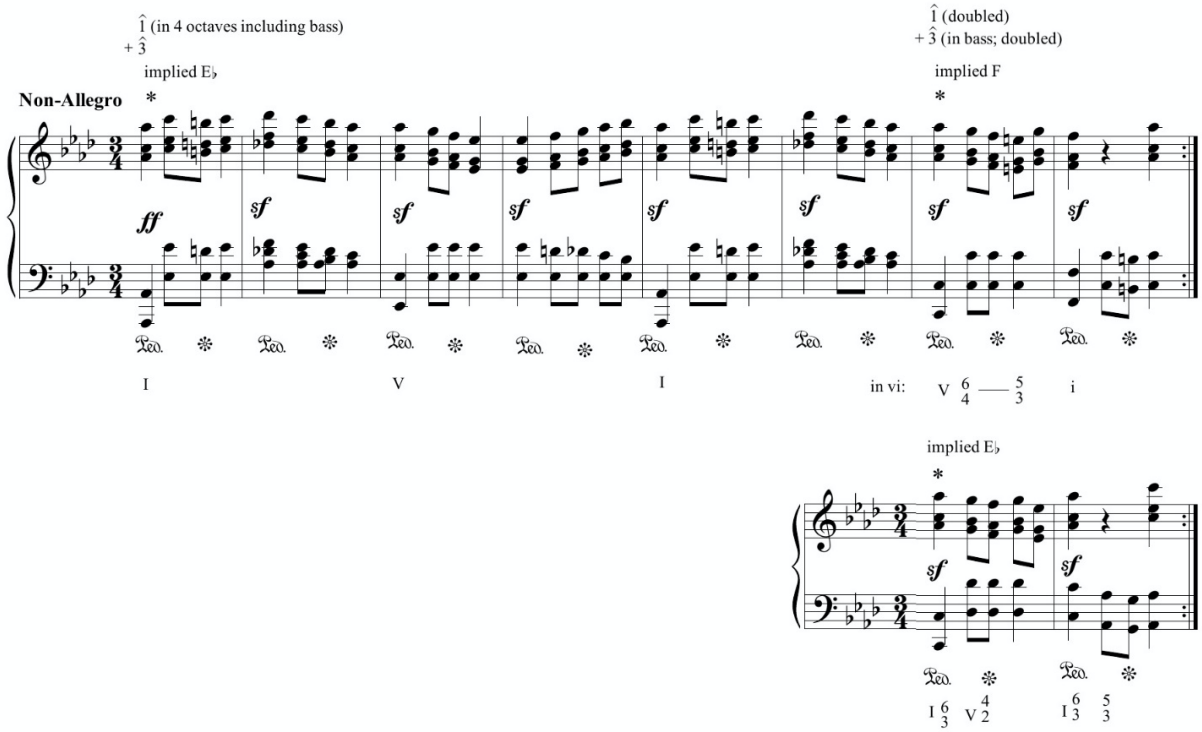
In the hypothetical alternative in Example 6, the music cadences in G major with no modulation. In relation to the father's response in B minor, however, the same D+B♮ dyad is already a firm tonic in the new key, with F♯ rather than G as the implied third member of the harmony.[12] Unlike the momentary pivot dyad in Example 4 (Bach's Invention), the pivot dyad in the *Erlkönig* endures more than a complete measure, which might direct listeners to the bare sound of the dyad. There is sufficient time to imagine a change in the filling in of the dyad into a complete chord, but this would not be expected in real-time listening (perhaps only in repeated listening when one is aware of the following change).

A hermeneutic reading comes to mind: what the child and the father actually hear is the same, but what they conceive is very different, each of them adding different elements to the sensory input that sum up to a different understanding of the whole situation. The child's implied chord is major, as is the seductive music in the stanzas sung by the erl-king, while the father's implied chord is minor, serious, and inattentive to these seductions. Notice that the roles of "optimistic" major and "pessimistic" minor have been reversed.

Example 7 includes a tentative pivot dyad of a different kind. This is a parallel period whose consequent modulates from the tonic A♭ major to the submediant. The chord on the strong beat of the penultimate measure is, in retrospect, a cadential 6/4 in F minor.

[12] The end of stanza 4 reverts indeed to G major, the goal of the hypothetical recomposition.

EXAMPLE 7. Schumann, *Carnaval*, piece No. 21, "Marche des Davidsbündler contre les Philistins," mm. 1–8. $\hat{1}$+$\hat{3}$ as I6/3 or cadential 6/4 in vi (leading to cadence in modulatory consequent)? Annotated score with hypothetical non-modulatory recomposition.



The proposed fourth of this cadential six-four, the tone F, arrives at the next beat after a foreground unfolding. On the first beat, F is still implied. This is our first encounter with a dissonant implied tone, a paradoxical phenomenon that Rothstein (1991) hardly acknowledges in his seminal study of implied tones.[13] The dyad at that moment consists of A♭ and C alone, and has the same pitch class content and the same texture as the opening sonority, but not the same bass and same doublings. In real-time listening, the dyad that stands for the cadential six-four in F minor might have represented a non-modulatory tonic in first inversion, with implied E♭ rather than F, as

[13] Glimpses of dissonant implied tones in Rothstein (1991): his example 16 (from Schumann, Piano Quintet, i) starts with V7, where the bass is left off but is retained conceptually when its upper dyad (fifth and seventh of V7) sounds alone; in his example 17 (from Bach's Violin Sonata BWV 1001, iv), m. 12, a 4–3 arpeggiated suspension becomes simultaneous in the imaginary continuo. The reduction in his example 19 is rejected due to the vertical dissonances.

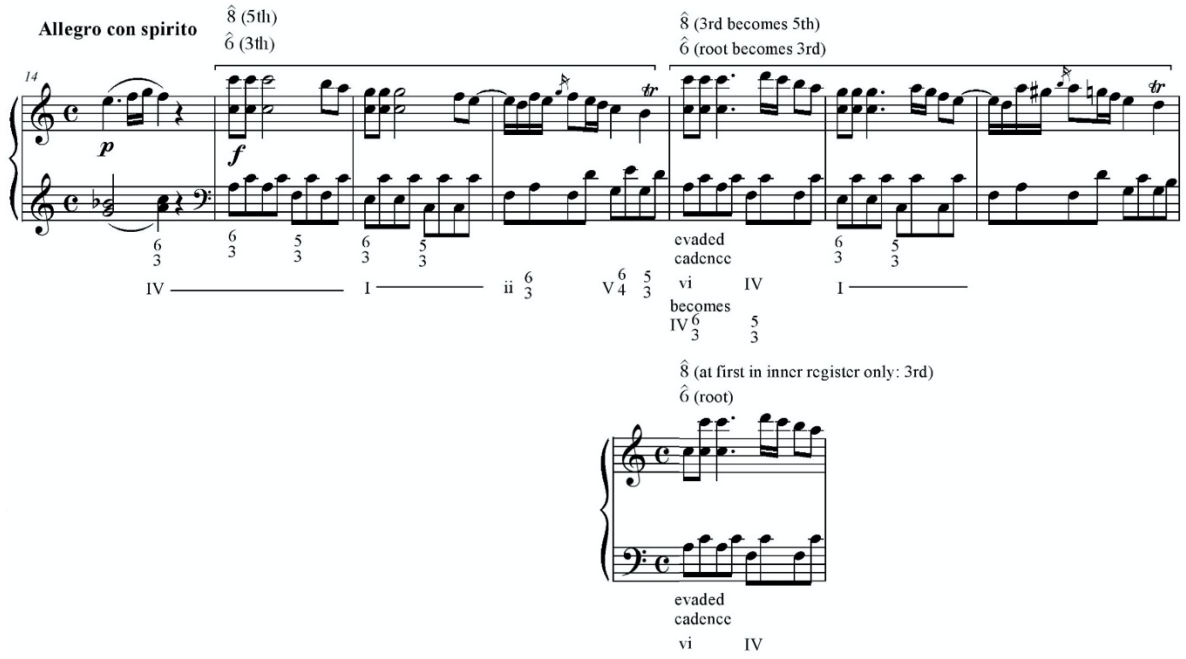
shown in the lower set of systems of Example 7. Had the music stopped after the first beat of m. 7, there would still have been no sufficient clues at that moment for the modulation that happens immediately after it.[14] This is a weak kind of pivot, since its unrealized meaning is only tentative even in real-time listening. There are indeed analytical arguments in favor of a cadential 6/4 already at the downbeat of m. 7: the parallelism with the cadential 6/4 in the main key in m. 3, and the harmonic rhythm of one chord per bar. However, even if conceived as a cadential 6/4 only, the dyad in Example 7, m. 7 has relations of reinterpretation with the opening dyad of the piece, as these two dyads share a similar voicing with invariant right hand.

§3 THIRD-RELATED READINGS OF AMBIGUOUS THIRDS IN OTHER LOCATIONS IN THE KEY

Up to now, all the dyad thirds we have encountered were members of the tonic (or at least of a tonicized chord) that could also represent the mediant or submediant. However, such thirds can also take place in other locations in the key. Example 8 is a tentative case in point.

[14] The cadential moment has been significantly weakened in the recomposition. The bass of the C+A♭ dyad in its new context as I6/3 might have continued C–D♭–E♭ and arrive at a root-position V in the manner of an expanded cadential progression (as discussed in Caplin 1987). This would create parallel fifths D♭/A♭–E♭/B♭ with an inner voice, then contrary octaves E♭–A♭ with the upper voice, unless the upper voice is modified.

EXAMPLE 8. Mozart, Piano Sonata in C major, K. 309, i, mm. 14–20. $\hat{6}$+$\hat{1}$ ($\hat{8}$) as third and fifth of IV reinterpreted as root and third of vi (tentative): Annotated score with hypothetical recomposition reinforcing the sense of deceptive cadence



As in Example 3, the passage from K. 309 has a potential change of meaning when the music recurs, but now the repeated section is a mere three-measure module (see brackets). The first appearance of the dyad $\hat{6}$+$\hat{8}$ (m. 15) is a clear IV of C major, arriving after a tonicized IV has already been heard, and proceeding via downward chordal skip to the root of IV. The repeat at m. 18 is more complex. This is a moment of expected ending (after a root-position V with a trill), where $\hat{6}$+$\hat{8}$ would stand for a deceptive cadence, as incomplete vi.[15] However, the same dyad marks a renewed beginning of a three-measure module, a genuine evaded cadence of the "one more time" type (as Schmalfeldt [1992] explains). Schmalfeldt (p. 14) distinguishes such genuine evaded cadences where "there [is] simply no ending" from an elision, which "simultaneously marks both the end of the phrase and the beginning of the next." Applying this distinction to the passage from K. 309, one might find at m. 18 a genuine evaded cadence, a new beginning only (and hence IV6/3

[15] IV6/3 can serve in deceptive cadences, but as the dyad $\hat{6}$+$\hat{8}$ sounds alone, the inferred harmony from the previous activity is certainly the usual vi.

only), supported by a registral leap. The sense of elided deceptive cadence (vi that becomes IV6/3) could have been much stronger had the leading tone resolved first in the inner register only (as in the hypothetical alternative in Example 8).[16]

However, the actual passage does not cancel out the sense of potential deceptive cadence. In ordinary elisions as well as genuine evaded cadences, the identity of the chord at the juncture is unequivocal (the tonic).[17] By contrast, in K. 309 the potential ending and new beginning share a dyad but require different implied tones, and thus give rise to different chords. Even if the new beginning overrides the potential deceptive cadence, it does not eradicate its presence altogether.

In tonal music, the way dyads serve third-relations depends on their location within the key. Diatonic interpretation of dyads must of course include diminished triads, which stray from the consonant framework presented above in Example 2. Appendix 2 shows all diatonic dyads and all diatonic triads that include them.

Even without ambiguity, a dyad may imply a dissonance simply due to the diatonic framework.[18] More pertinent to our inquiry is the priority of the main, prototypical scale degrees. Consider Example 9.[19]

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[16] Schmalfeldt (1992, 36) shows a very similar case from Beethoven, Cello Sonata, Op. 69, iii. The evaded cadence at m. 199 is on IV6/3. In the actual music, the melody does close in the inner register before leaping to the renewed beginning in the upper register, thus providing a sense of elision. The IV6/3 serves as a deceptive resolution to the preceding cadence, but since in that example the chord is complete, no sense of vi is invoked.

[17] Evaded cadences on the tonic are the only type of evaded cadences Caplin (1998, 101–106) shows. Schmalfeldt (1992), by contrast, also regards evaded cadences of the "one more time" type with a predominant harmony as the first chord in the renewed statement, most notably in her example 1b (♭II in Chopin, Etude, Op. 10, No. 6, m. 49) and example 7 (V6/3 of V in Cherubino's aria "non so più" from *Le Nozze di Figaro*, m. 31).

[18] See the penultimate measure of Scarlatti, sonata in C minor K.116, L.452: the dyad $\hat{2}$ above $\hat{4}$ in the bass before V serves as the dissonant but diatonic ii°6/3. Mozart prohibited such a sixth on an accented beat in a two-part counterpoint exercise. On this exceptional comment, see Lester 2016, 243.

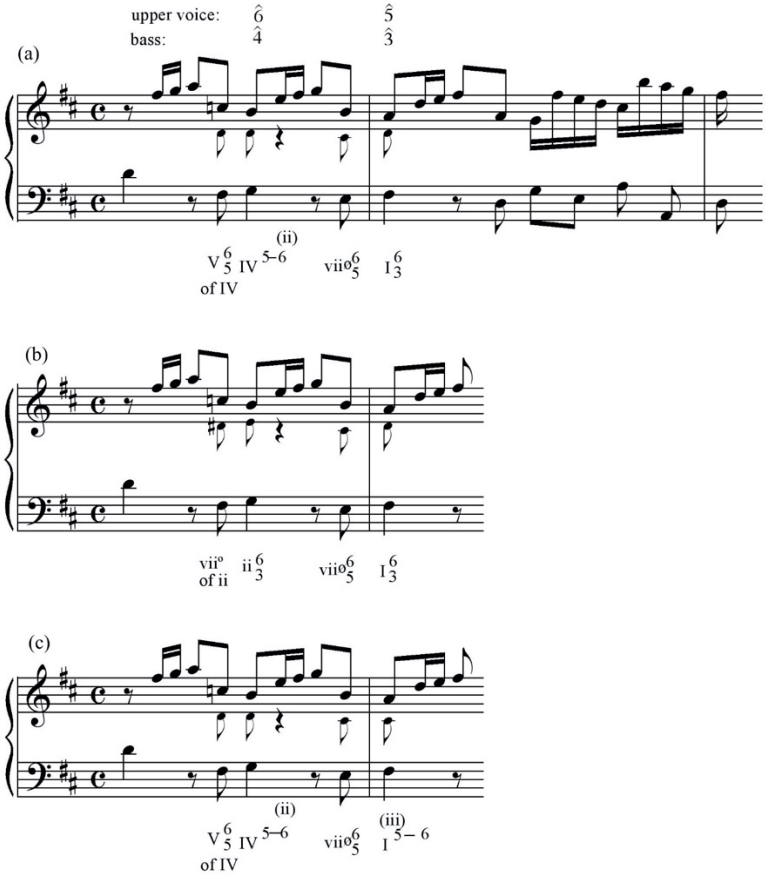
[19] For a different realization of the excerpt from the Sinfonia, involving suspension tones, see Christ et al., 1966, 96.

EXAMPLE 9. Bach, Sinfonia in D major, BWV 789, mm. 1–2. Implied inner voice added in small notes. Three different harmonic interpretations of the dyad succession (4̂+6̂)–(3̂+5̂).

(a) Inconsistent but preferred reading: 4̂+6̂ as IV5–[ii]6, 3̂+5̂ as I6/3 from the start;

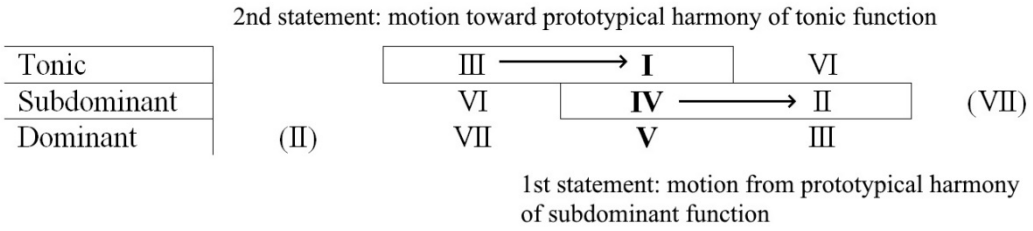
(b) Consistent harmonization with leading-tone 6/5 chords;

(c) Consistent harmonization with 5–6 motion.



The example is the two-part opening of a three-part piece. The theme exhibits a melodic sequence in both upper voice and bass, but the default harmonization of the sequential statements is arguably not sequential. Example 9a shows the way I find to be the convincing harmonization. The first statement leads to a root-position triad on IV, after which comes a contrapuntal 5 to 6 motion, while the second statement arrives at the tonic in first inversion without change of harmony in the weak part of the beat. Examples 9b and 9c show alternative harmonizations that treat the melodic sequence consistently. In Example 9b, both statements arrive at a 6/3 chord. This is indeed possible, but the need to use a raised chromatic tone makes this harmonization less plausible. The harmonization in Example 9c is more problematic. The iii chord with 5 to 6 change into first-inversion I is a sequential duplication of the IV chord with 5 to 6 change into first-inversion ii. Nevertheless, the hierarchy between the chords is not the same in each pair; see Example 10 after Eytan Agmon.

EXAMPLE 10: Explanation of difference in preferences in Example 9a, after ideas by Agmon (1995, 201).



In the former statement, the motion from IV to ii, the initial chord is the prototypical representative of the subdominant function, but in the latter statement, moving from iii to I, it is the goal that is prototypical of the function. Thus, even in the actual surface sequence IV–ii, iii–I, the reduction would be IV–I. The sequence proceeds to a third statement. The status of the I6/3 might be passing, in support of a passing tone A in the melody, connecting B (part of IV) and G (part of ii). This does not affect the preference of I over iii at the surface level.

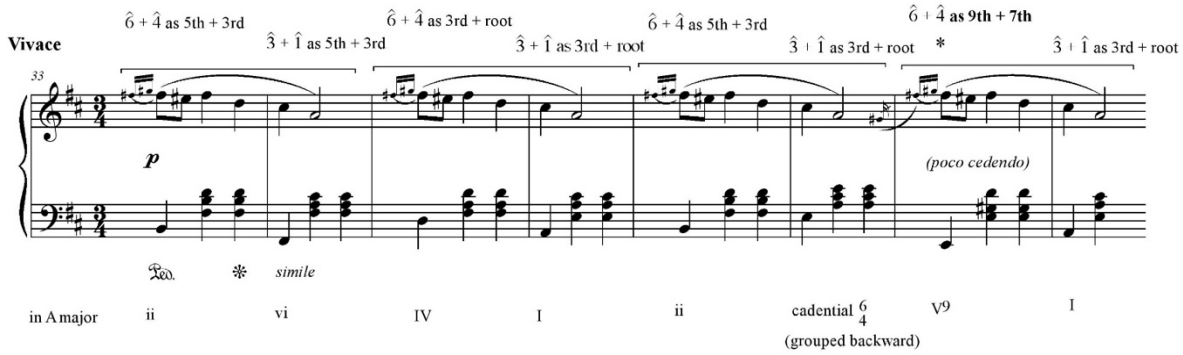
Beyond the diatonic options, an exhaustive examination of all possible ways that consonant thirds can be embedded in triads should acknowledge chromatic dyads (that involve at least one non-diatonic tone) as well as chromatic triads even for diatonic thirds (e.g., ♭VII as the framework of $\hat{2}$+$\hat{4}$). See Appendix 3. All scale degrees are demonstrated in relation to C major or minor (natural or harmonic). The table includes all major and minor thirds. It does not include enharmonic interpretations (augmented seconds for minor thirds), to be discussed in §7. However, it turns out that some dyads would be interpreted across the enharmonic seam, depending on whether they serve as a lower or upper dyad of a triad. Thus, semitones 4 and 7 above C would count as E♭ and G♭ when they are the lower dyad of a minor triad but as D♯ and F♯ when they are the upper dyad of a major triad. The discrepancy between diatonic triads and consonant triads also comes to the fore. Major thirds are unlikely to be interpreted as augmented triads, at least not without special context. Minor thirds, by contrast, do lend themselves also to interpretation as part of diminished triads. In Appendix 3, I have made a compromise: the diatonic diminished triads are shown (in bold font), but other diminished triads that would be secondary leading-tone triads are avoided.

§4 UNACCOMPANIED THIRDS INTERPRETED AS UPPER PARTIALS OF STACKS OF THIRDS

Up to now, all the dyad thirds were completed into basic triads: they paired either roots and thirds that gave rise to implied fifths, or thirds and fifths that gave rise to implied roots. It might seem unlikely that, on hearing consonant thirds, listeners would interpret them as upper partials of seventh chords and beyond. Certain examples, however, strongly suggest just this.

Of course, when dyads are harmonized explicitly, it is not a problem to make them a combination of a fifth and seventh or seventh and ninth (if not also of higher members of stacks of thirds). For an instance with an especially salient dyad, see Example 11. The passage is based on a quadruple recurrence of two melodic dyads ($\hat{6}$ to $\hat{4}$ and $\hat{3}$ to $\hat{1}$ in the local key A major; one dyad per measure) with constant reharmonizations. In the last statement, the former dyad becomes the seventh and ninth of V9.[20]

EXAMPLE 11. Chopin, Mazurka Op. 30, No. 2, mm. 33–40. Different harmonizations of recurring dyads, including higher elements of stacks of thirds.



[20] In Example 11, the third statement of the $\hat{3}$+$\hat{1}$ dyad reaches a cadential six-four chord that is motivically grouped backward rather than with its resolution into V5/3. See Goldenberg (2017, 41–49) on that phenomenon.

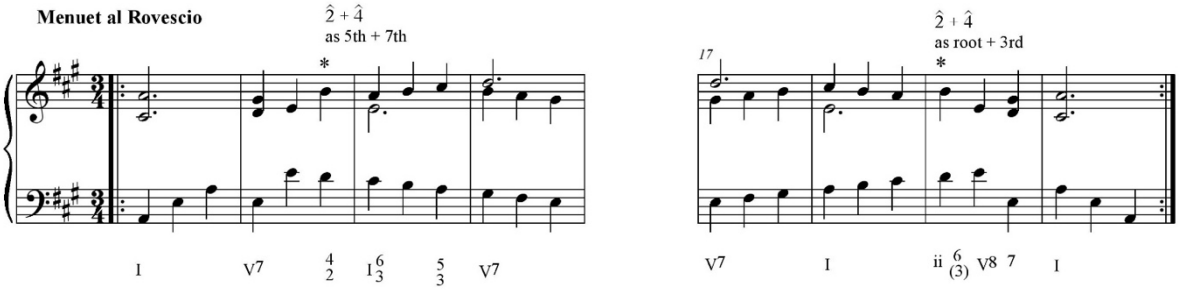
Out of context, it is indeed unlikely that unaccompanied consonant thirds would be perceived as part of dissonant chords. A simultaneous dyad may, however, sound as involving a seventh or even a ninth due to the perceptual retention of the preceding tones. Consider Example 12, from a minuet in retrograde by Haydn. Heretofore, we have not met this kind of interpretive change involving a repetition. Since the direction of tonal motion does not go backward, Haydn had to find devices that enable reinterpretation. As Robert Morgan (1998, 26) observed, one of these devices is the use of incomplete texture: "the omission of the inner voices of the V4/2 at m. 2 enables it to become a dominant preparation (ii6) at m. 19."[21]

In the first occurrence of the $\hat{2}$+$\hat{4}$ dyad (m. 2), $\hat{4}$ thus functions as a dissonant element (a seventh) even though only a consonant third is played at the same moment. At that moment, neither the root nor the third of V7 are played, but the root persists from the beat before.[22]

[21] Morgan describes the V4/2 and ii6 as real objects. A possible meta-theoretical objection would prefer to regard the actual dyad as the real object, stating that the use of two parts only enables the same tones to be interpreted as both V4/2 and ii6.

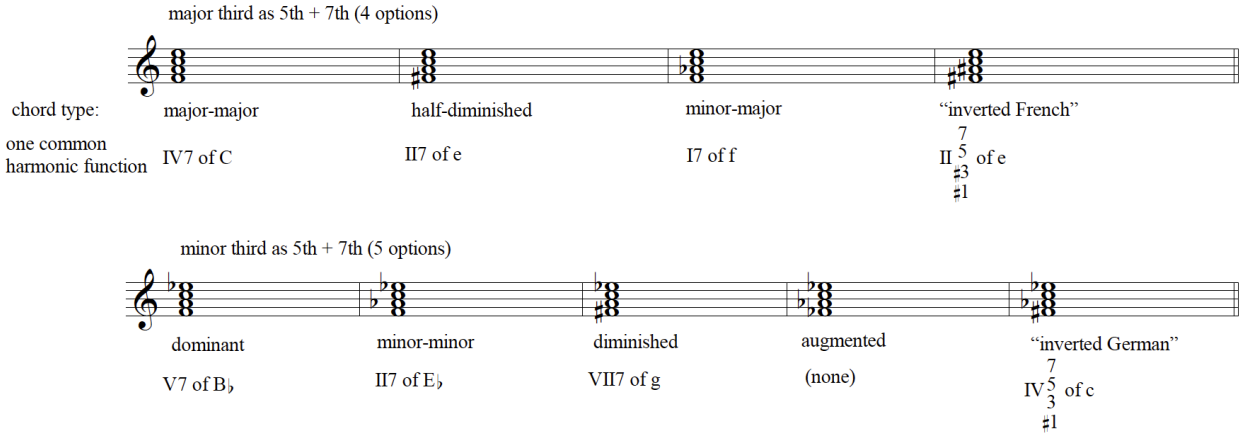
[22] For a different scenario of $\hat{2}$+$\hat{4}$ as part of ii or V7, see Schubert, Piano Sonata, D. 850, ii, m. 16, third beat. Is this an arpeggiated dyad that represents ii, the resolution of its secondary dominant in the preceding measure (as Damschroder [2010, 111] reads)? Or, alternatively, does it merge with $\hat{5}$ from beats 1 and 2, making V4/3, in which case the $\sharp\hat{1}$+$\sharp\hat{3}$ in the preceding measure are passing chromatic tones towards the fifth and seventh of the inverted V7?

EXAMPLE 12. Haydn, Piano Sonata, Hob. XVI:26, ii (=Symphony No. 47, iii). $\hat{2}$+$\hat{4}$ as root and third of ii or fifth and seventh of V7: mm. 1–4 (original form) vs. mm. 17–20 (retrograde).



In principle, seventh chords other than V7 may also be presented by their upper consonant third alone. Example 13 presents these options schematically (with one representative harmony for each seventh chord type).

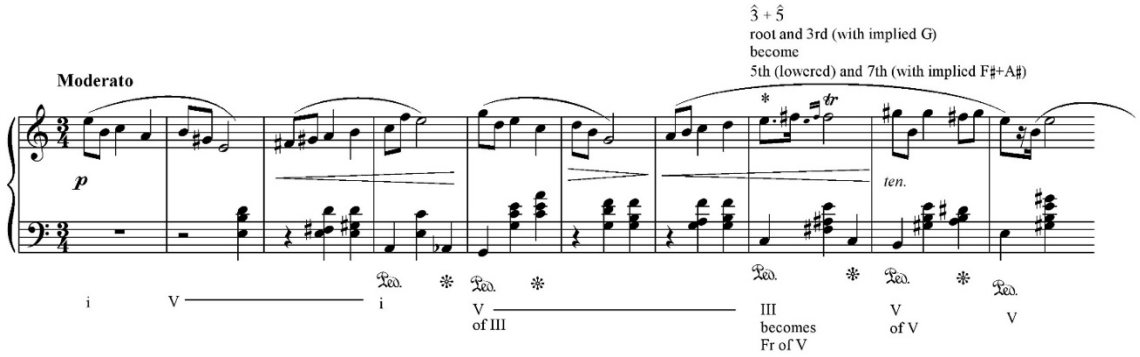
EXAMPLE 13. Consonant thirds as upper members of seventh chords.



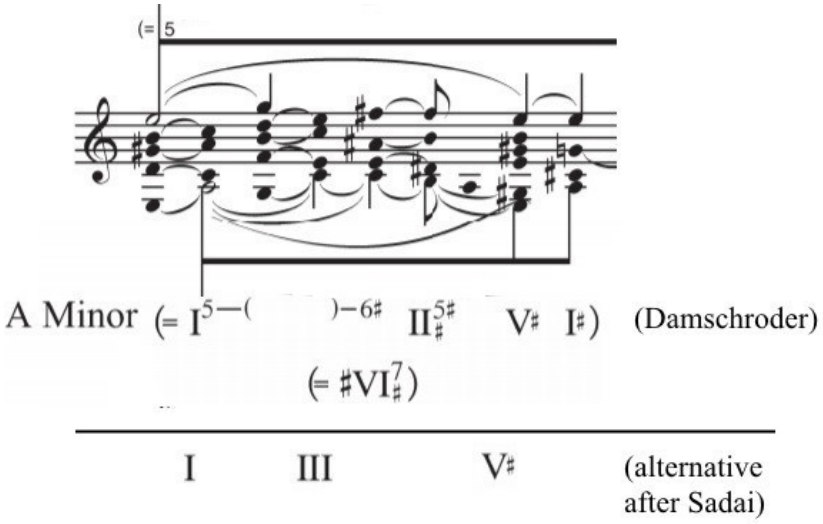
The options presented in Example 13 include along with the diatonic seventh chords also chords with diminished thirds, more familiar in inverted form as augmented-sixth chords. Even though it might seem unlikely that a consonant third would imply a chord that includes a diminished third, or even involve ambiguity where one of the involved chords includes a diminished third, this does happen in Example 14.

EXAMPLE 14. Chopin, Mazurka. Op. 59, No. 1, mm. 1–10. $\hat{3}$+$\hat{5}$ as root and third of III or already as fifth and seventh of French II♯6/5 of V?

(a) Annotated score



(b) Analysis from Damschroder (2015, 139, Example 3.16) © David Damschroder 2015, published by Cambridge University Press, with alternative after Sadai (1980, 218, Example 473).



Notice the dyad $\hat{3}$+$\hat{5}$ (C+E) at the downbeat of m. 8. As in Example 4 above, the preceding and ensuing contexts provide conflicting clues for a mid-phrase dyad. It arrives as a clear III, a

resolution of V/III that lasts for three measures (a cadential 6/4 that proceeds to V7). The continuation from that dyad makes it the fifth and seventh of a French augmented-sixth chord: the harmonic rhythm is one chord per measure, and the second beat is an arpeggiation from the first beat, as it was in m. 4. While the actual tones in beat 2 of m. 8 might have represented V7 of V/V without the fifth, that missing fifth would be C♯—instead of which the third beat returns to the bass and uses C♮.[23]

In this case, the view of the harmonic hierarchy in the theme as a whole might influence the manner one reads the dyad, as shown by the summary of analyses in Example 14b. Yitzhak Sadai, in a common-sense way, notes the intermediate tonicization of the mediant, C major, on the way from I to V.[24] Damschroder, by contrast, deliberately undermines the mediant as passing toward the augmented sixth in a 5–6♯ motion. A reading that interprets the French augmented-sixth chord already at the downbeat of m. 8 would fit better Damschroder's conception of the passage.[25]

Could unaccompanied thirds represent even higher members of stacks of thirds, i.e., seventh and ninth, if not beyond? This requires the omission of multiple low chord members, an idea that has parallels concerning fuller texture. Extending the familiar tradition that regards the viiº triad as an incomplete (rootless) form of V7,[26] Prout (1889/1903, 169 and 181) argues that II and IV are incomplete forms of V9 and V11 respectively: "If both the root and third are omitted in the chord of the dominant major ninth, the remaining notes (the fifth, the seventh and the ninth) form a diatonic chord on the supertonic of the major key" and "If the generator, third, and fifth of the chord [of the dominant eleventh] are all absent, the seventh, ninth, and eleventh give the triad

[23] If the French chord (C/E/F♯/A♯) in the present context is read like an alteration of C♯/E/F♯/A♯, (V4/3 of V of E major), the bass tone C♮ itself is an altered tone (V6/4/**3**/♭**1** of V, the members of the explicit dyad C+E shown in bold font). In E minor (minor V) the bass C♮ would have been diatonic (a member of C/E/F♯/A, the diatonic II of E minor), with the French chord as II♯6/4/**3**/**1**), but since the goal is E major, the diatonic version of II would be C♯/E/F♯/A, so C♮ is an altered tone also when conceived as altered II (♯6/4/**3**/♭**1**) rather than altered V/V.

[24] Sadai writes the names of the keys (A minor, C major, E major) below the score.

[25] On tonicizations contradicted by voice-leading continuity, see Schachter 1987, 298.

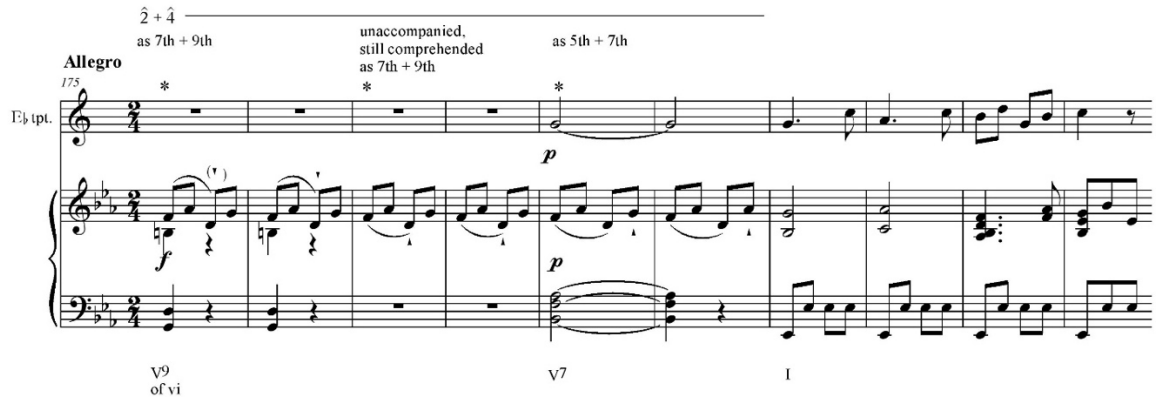
[26] That idea can be traced back to Kirnberger. Sechter expanded it to other scale degrees as well. See Wason 1983, 55–57.

on the subdominant."[27] Prout seems to push the omission of lower tone members *ad absurdum*. Notice, however, Example 15 where a dyad third serves as the seventh and ninth of V9 (and, also, Example 21 below for a dyad fifth as the fifth and ninth of V9).

In Example 15, the lower members of a secondary V9 are left off, letting the seventh and ninth of that chord sound alone. The dissonant context remains, even when we hear the consonant interval only. In each measure, when the bass of V9 is present as well as when the upper dyads remain alone, the ninth resolves at the weak beat as a 9–8 appoggiatura. When the accompaniment returns, the invariant tones become the fifth and seventh of V7, and since the A♭ in the accompaniment continues explicitly, the hierarchy between A♭ and G in the upper part is reversed (G becomes a lower neighbor to A♭).

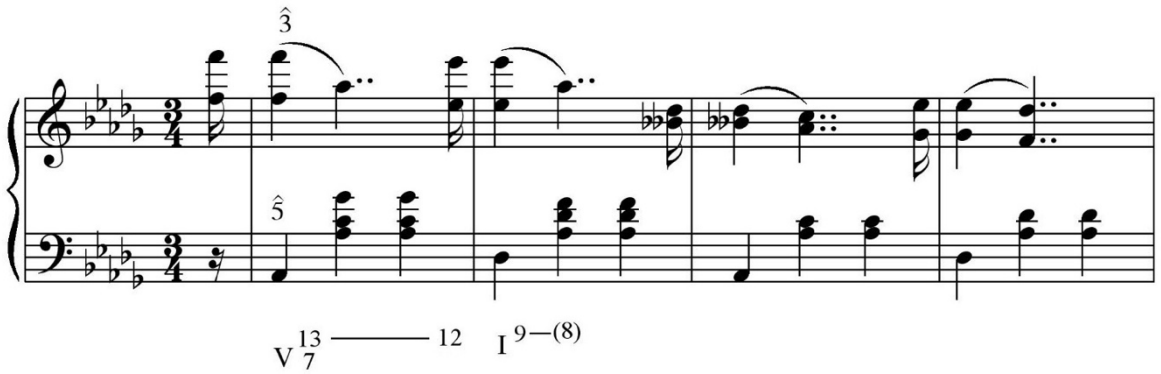
[27] Damschroder (2008, 13) notes that already in 1798, in Johann Gottlieb Portmann's practice, "the root, and even [also] the third, may be omitted."

EXAMPLE 15. Haydn, Trumpet Concerto, iii, mm. 175–84. $\hat{4}$+$\hat{6}$ as fifth and seventh of V7 or seventh and ninth of V9/vi



In Example 16, the opening dyad (a compound sixth combining $\hat{3}$ and, in the bass, $\hat{5}$) consists of the root and the uppermost member of V13/7 (It could also be conceived as a 6–5 non-harmonic tone). The third and the seventh enter on the next beat, but tellingly the exceptional meaning of the dyad is clear from the outset. The reason is the extreme registral contrast of the dyad: $A♭^2$ in the bass with F^5+F^6 (doubled) in the upper voice more than two octaves apart. In this voicing, V13/7 of D♭ is more expected a harmonization than an incomplete F minor 6/3 or D♭ major 6/4.

EXAMPLE 16. Schubert, Ländler, D.366, No. 14. $\hat{3}$ over $\hat{5}$ as incomplete V13/7.

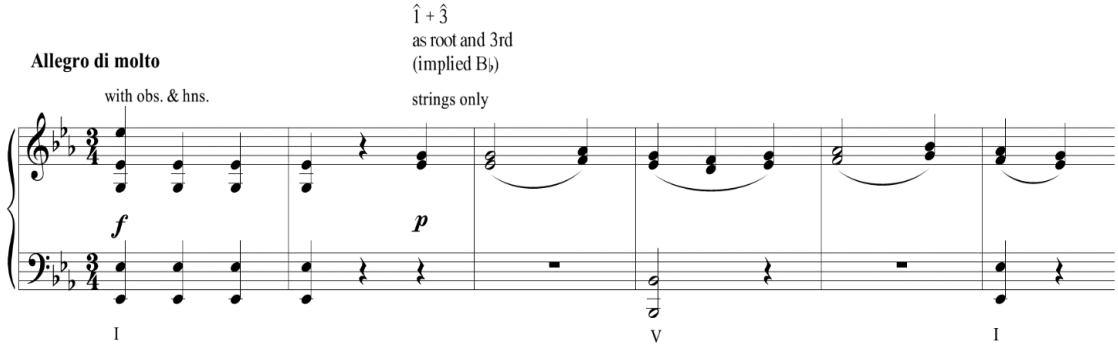


§5 DISSONANT TONES IMPLIED BY THIRDS THAT ARE TRIAD MEMBERS

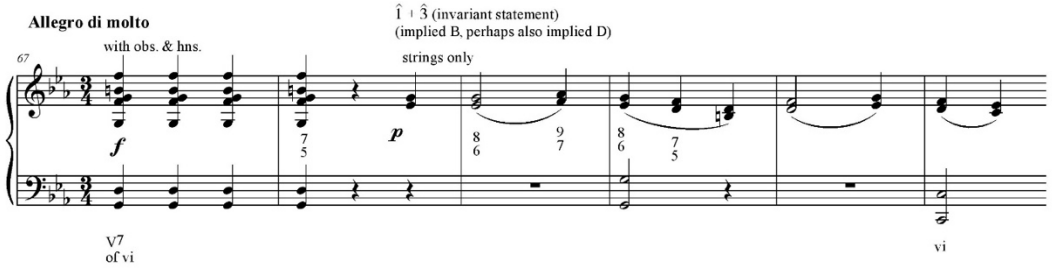
Even when an unaccompanied third does belong to a basic triad, the implied tone may be both dissonant and chromatic.[28] Consider Example 17. The first theme starts in incomplete texture after introductory full chords. In the exposition (Example 17a), the context is normative, and the dyad serves as the root and third of a stable tonic.

EXAMPLE 17. Haydn, Symphony No. 55, i. $\hat{1}$+$\hat{3}$ as root and third of I become members of a passing dissonance.

(a) Beginning of exposition.



(b) Beginning of development.



[28] Diatonic dissonant implied tones are normative. See note 18 above.

The third measure in the development (Example 17b) is identical—including instrumentation—to that of the exposition, but the difference in tonal context changes the implied chord. This is not a case of a reinterpretation of an exact repetition like Example 3, but rather a related and more intricate artistic device, i.e., a reinterpretation of invariant analogous moments within larger rotations.

In the exposition, the E♭+G ($\hat{1}$+$\hat{3}$) dyad is part of the tonic, and the implied tone B♭ does not involve any complication whatsoever. The tones of the F+A♭ dyad on the third beat are clearly foreign, best understood as incomplete upper neighbors that anticipate the V in the next measure. The exact status of A♭ at beat 3 is intricate: does it fade out or retain as the seventh of V in the next measure?

In the development, the E♭+G dyad is surrounded by V7 of vi. The implied tone is B♮; the implied passing chord might be conceived as an augmented triad, or, arguably, the D in the lower register also remains implied. There are two related ways to understand the details: either the E♭+G dyad serves as a pair of complete upper neighbors to D+F (with the F+A♭ as a surface incomplete upper neighbor, much like in the exposition), or E♭+G serve as pair of passing tones V7/5–8/6–9/7 toward F+A♭. In that case, the status of F+A♭ changes into chordal members of V9 of vi.

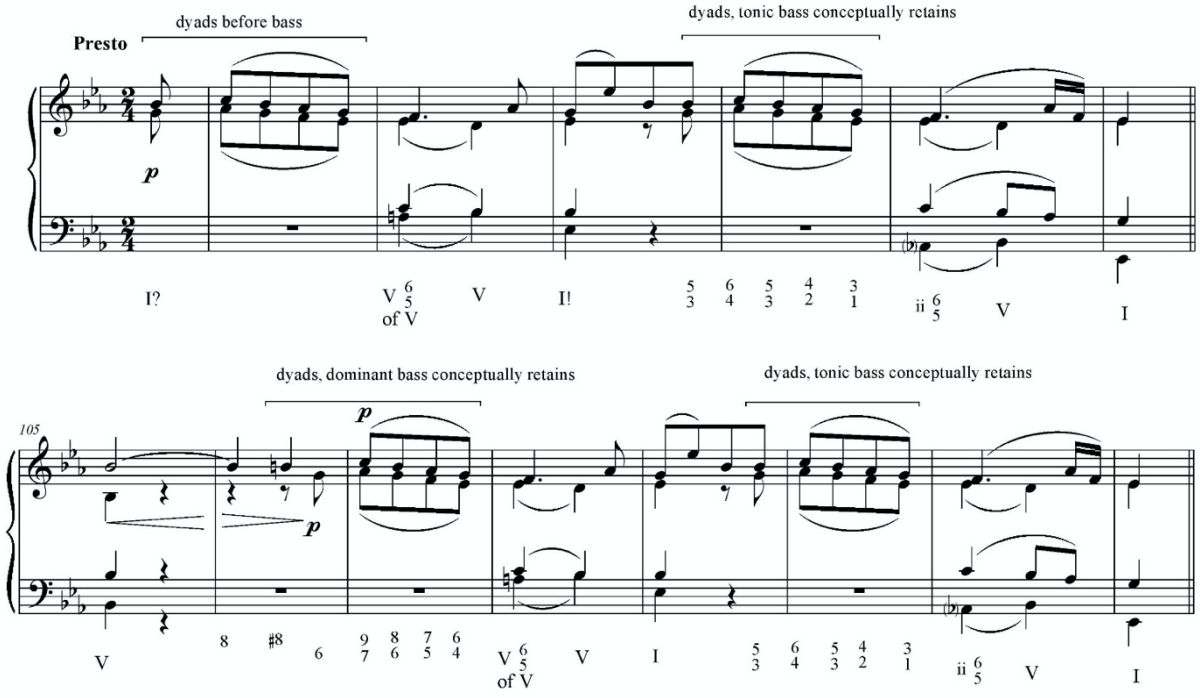
The change arguably affects the feeling of hypermeter in the theme. In the exposition, the hypermeter of the upper parts and the bass are not synchronized.[29] In the bass, mm. 4 and 6 are strong measures while mm. 3 and 5 are silent, but in the upper parts m. 3 is a new beginning and a strong beat. The structure of mm. 3–6 is complicated by a built-in non-concurrence between harmony and motive. This is a case of I–V, V–I statement–response repetition (as explained in Caplin 1998, 39). The motivic pairing of mm. 3–4, 5–6 makes m. 5 a strong measure (in the upper voice) in parallel to m. 3, while the harmony connects mm. 3 and 6. In the development, the stable element of the third measure disappears, as the content of that measure becomes unstable.

[29] On conflicting metrical patterns, see Kamien 1993.

The interpretive change from consonant to dissonant dyad serves an aesthetic goal of intensification paradigmatic of development sections. A dissonant deformation of the first theme is a rare option to begin development sections. Hepokoski and Darcy (2006) do not mention this option, but several examples in the literature use it.[30] What is extraordinary about Haydn's Symphony No. 55 is that the dissonant deformation occurs in the listeners' conceptual completion of identical auditory data.

In certain circumstances, the implied tones that complete the dyads need not be part of the harmony. When a strong-beat tone is left off, especially in the bass, it retains as default within the imagery continuo, letting faster thirds sound as dissonances in relation to it. The incomplete texture of dyads may also lend itself to changing bass tones. See Example 18.

EXAMPLE 18. Haydn, String Quartet, Op. 20, No. 1, iv, mm. 1–6 and 105–112. thirds as non-harmonic elements over an imaginary continuo with added figured bass.



[30] Examples include Beethoven, Piano Sonata Op. 31, No. 2, iii, and Dvořák, "American" String Quartet, Op. 96, i.

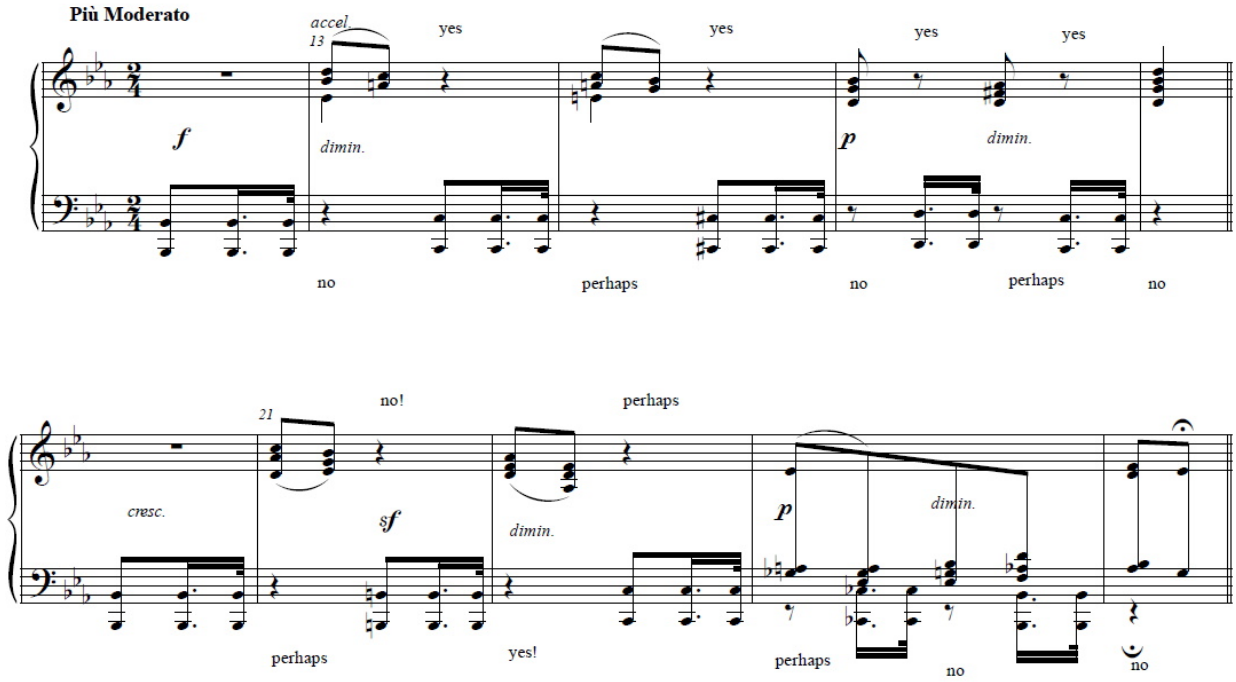
The first part of Example 18 shows the beginning of the theme of Haydn's movement. The violins play the series of dyads for the first time before any bass tone has been heard. An implied tonic is suggested but not yet certain. After a cadence of limited cadential scope (using the term coined by Caplin [2004, 86]), the bass of the tonic is left off but conceptually retains throughout the dyad measure, even when the dyads are dissonant in relation to that bass. In the retransition (second part of the example), it is the dominant that precedes the dyads. The new bass also retains conceptually, and creates different intervals with the dyads. The dyads are repeated after the tonic in the context already familiar from the exposition.[31]

Tones that exist explicitly but are then left off do not always retain during ensuing rests. The exact rules that dictate tone retention or lack thereof require further research, but it is possible to demonstrate the problem. See Example 19—we stray from the dyads to one example in fuller texture. The hands play alternately with many rests. The annotations "yes," "no," or "perhaps" indicate whether the left-off tones retain implied during the rests. Usually the tones or chords that preceded rests retain conceptually during unaccented rests but not during accented rests. Nevertheless, in mm. 21–22 there are exceptions in both directions, perhaps creating a sense of implicit syncopation.[32]

[31] The dyads over a conceptually retained dominant bass already occurred in the beginning of the development.

[32] On five occasions in Example 19, I could not decide whether the tones that precede the rest retain conceptually. In four of these cases, the alternative is that the tones following the rest already start conceptually at the beginning of the rest. In m. 21, the retention of B♭ in the bass would create a 6/4 chord. The alternative would be a genuine resolution into a root-position tonic.

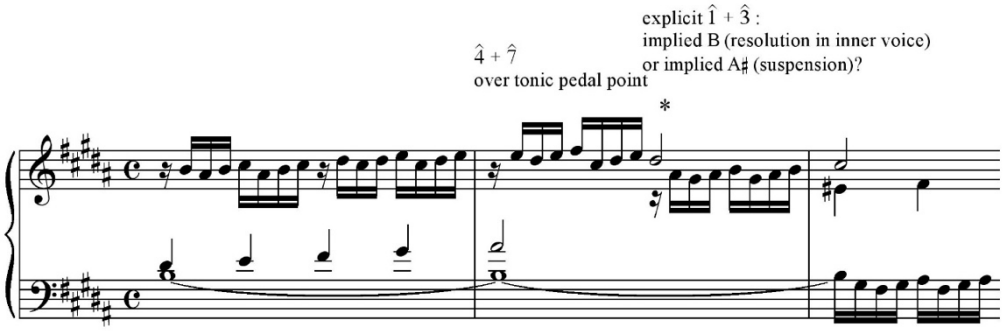
EXAMPLE 19. Mendelssohn, Variations Op. 82, variation no. 4, mm. 13–16 and 21–24. Demonstration of the difficulties in determining tone retention through rests (yes, no, or perhaps).



Example 20 shows an especially intricate case of an accented consonant third that implies a non-harmonic tone.

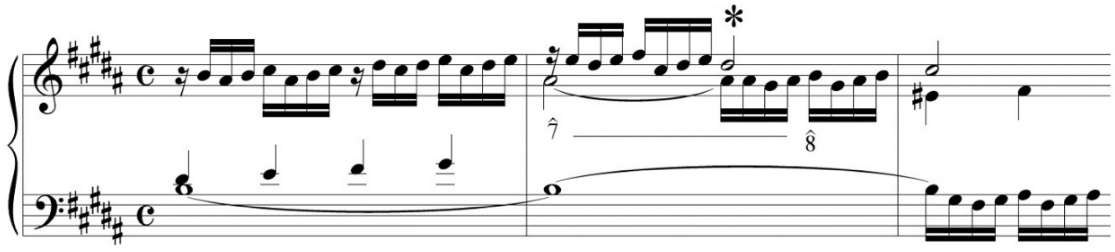
EXAMPLE 20. Bach, *The Well-Tempered Clavier*, Book 1, Prelude in B major, BWV 868, mm. 1–4. An implied non-harmonic tone.

(a) Annotated score.

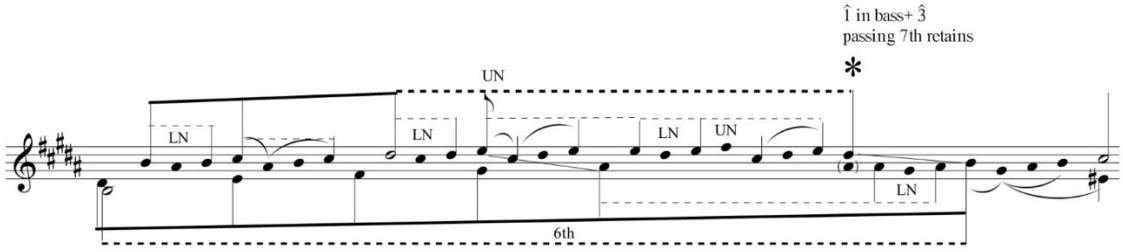


EXAMPLE 20 - continued

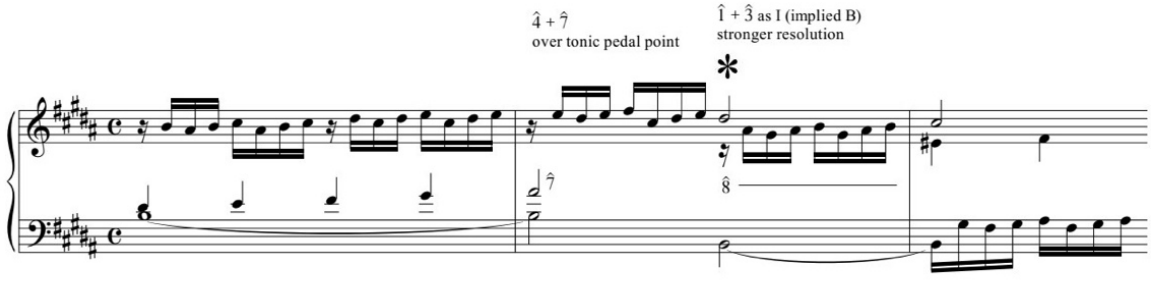
(b) Hypothetical variant with tied A♯.



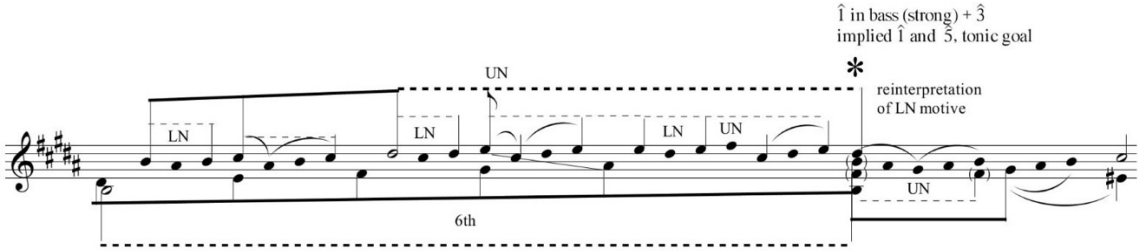
(c) Voice-leading graph.



(d) Hypothetical variant with implied resolution to consonance.



(e) Voice-leading graph of hypothetical version.



Notice in Example 20a the moment of rest in the inner part, at the middle of m. 2. Only B and D♯ (1̂+3̂) are present. One might assume a normative resolution of the preceding leading-tone tritone: E to D♯, A♯ to B. However, the bass below the mid-bar rest is hardly audible, as it has been held since the beginning of the piece, even without renewed attacks. As a result, an alternative way of hearing the dyad comes to the fore, in which the A♯ before the rest is retained conceptually during the rest as if it is tied (and continued in the explicit A♯ after the rest, Example 20b). In that reading, it is the dissonant seventh A♯ that is implied at the sixteenth-note that only the dyad B and D♯ is played. The very possibility of such a constellation is thought provoking, a warning sign against too simplistic theories for determining implied tones. Nevertheless, it is noteworthy that although this constellation of tones is very exceptional theoretically, it does not "jump to the ear" as it is sonically not harsh.

The retained A♯ is a step within the ascending sixth in the inner voice toward B on the fourth beat (Example 20c). A strong bass at that point, for example re-attacked at the lower register as in Example 20d, would modify the perception of the theme, and determine an implied B. In that alternative, the ascending sixth reaches its goal earlier, and, also, the following beat re-interprets the lower neighbor figuration (Example 20e).[33]

While the dyads that participate in non-harmonic combinations together with retained tones can themselves be passing tones (as in Example 18 above), the implied tones cannot be themselves passing and neighboring. Beyond common sense, I have currently no proof for that contention, except that I have found none and, also, failed to create such artificial configurations.

[33] A different reading: Beach 1997, 328. For other cases of implied suspensions, see Slottow (2005, 61) on Bach, Cello Suite No. 1, Minuet No. 2, mm. 1–4.

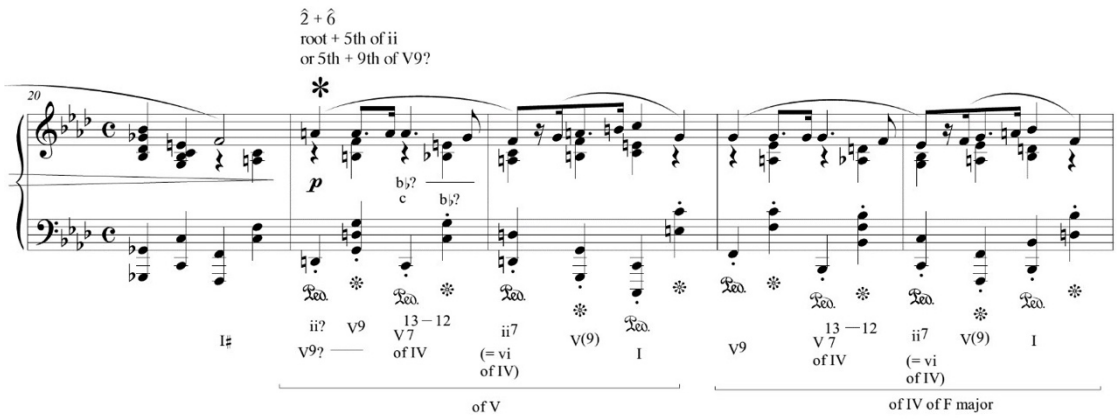
§6 UNACCOMPANIED DYADS OTHER THAN THIRDS AND SIXTHS

Occasionally, though probably not often, dyads other than thirds (or the complementary interval, sixths) lend themselves to more than one reading. We have already encountered one such case, in Example 9, where the diminished fifth F♯+C at the unaccented part of beat 2 is also subject to more than one reading: is this dyad a part of V7/IV (with implied D as in Example 9a) or of vii°7/ii (with implied D♯ as in Example 9b)? In both of these options the dyad members are the third and seventh of a seventh chord. Without any variant of D implied, the diminished fifth F♯+C would be the root and fifth of vii° of IV, as a mere triad (F♯, implied A, and C) or with a seventh (with implied E instead of D). The difference is mild, since vii° is of course wholly embedded within V7, as is vii°7 within V9.

In Example 21, the dyad on the downbeat of m. 21 is a perfect fifth, $\hat{2}$+$\hat{6}$, in the key of the dominant, where the added implied $\hat{4}$ is unequivocal (complete ii triad). At the next beat, the lower components of V9 enter (the bass above rather than below the bass of the incomplete ii). Should the dyad itself be read as the root and fifth of ii, or already as the fifth and ninth of the V9 that enters explicitly in the next beat?[34]

[34] In Example 21, beat 3 of the third measure is also a bare dyad, but that one is unequivocal.

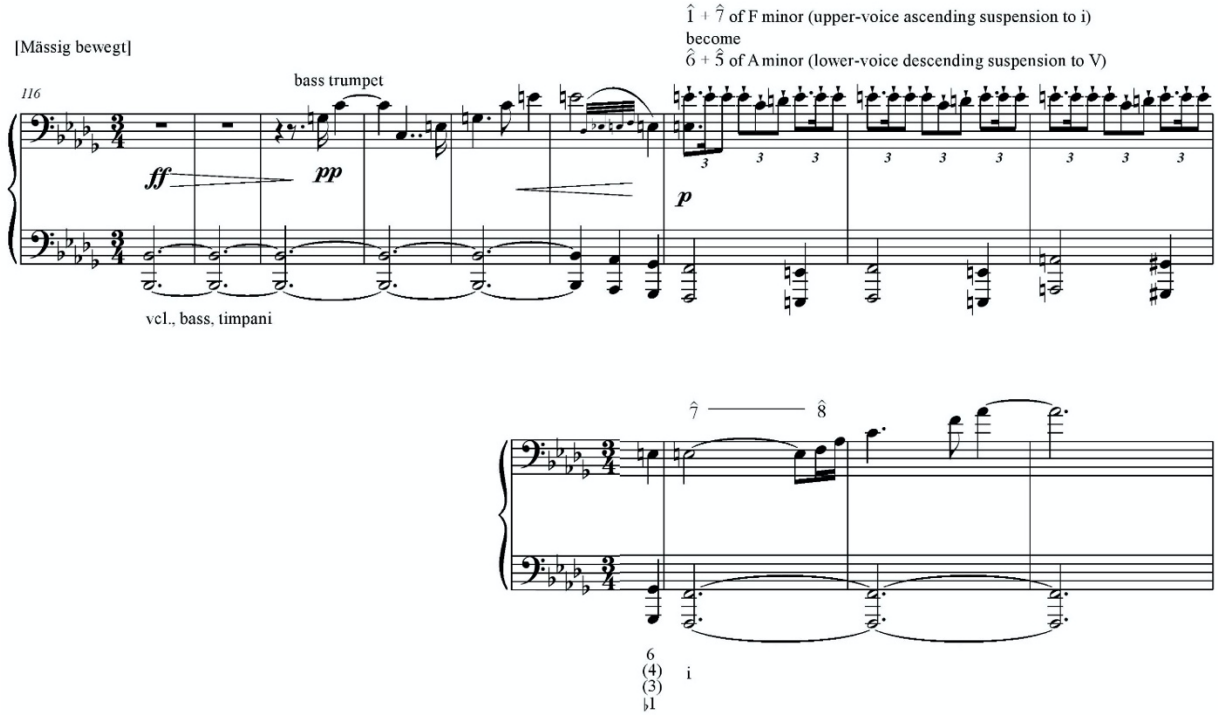
EXAMPLE 21. Chopin, Fantasy in F minor, Op. 49, mm. 20–24. Bare fifth ($\hat{2}$+$\hat{6}$) in the key of V as root and fifth of ii or fifth and ninth of V9.



Parallelisms in motivic content and harmonic rhythm give conflicting clues here. The sequential statement two measures later has an actual ninth on the downbeat of m. 23, and the harmonic rhythm there is clearly one chord per two beats. These features reinforce the sense of V9 already when we hear the consonant fifth D–A alone. On the other hand, m. 22 changes chords every beat and actually restates the ii–V progression.

In Example 22, the dyad is a major seventh that serves as a modulatory pivot.

EXAMPLE 22. Wagner, *Siegfried*, Prelude to Act 1, mm. 116–24. Bare major seventh as pivot: $\hat{1}$+$\hat{7}$ of F minor becomes $\hat{5}$+$\hat{6}$ of A minor. Annotated piano score, and hypothetical version in which the vertical seventh remains $\hat{1}$+$\hat{7}$.



Arriving from the inverted dominant seventh of F, the "Mime as smith" leitmotif enters with E as its main tone together with F in the bass. F in the bass becomes an appoggiatura to E, as $\hat{6}$ to $\hat{5}$ in A minor (the "servitude" leitmotif).[35] The new leitmotifs are so prevalent that it makes us forget that when E arrives above F its expected resolution would be F minor (as in the hypothetical version in Example 22). The pivot seventh F–E might have represented F/A/C/E, I7 in F major that becomes VI7 in A minor (without change in the implied complete chord). However, the passing A♭ (m. 121) in the bass on the way to F makes the implied key of potential resolution F minor, so that the dyad stands for F/A♭/C/E backward and becomes F/A/C/E forward.[36]

[35] The title "Mime as smith" comes from Donington 1963/74, 294 (motive D50). The "servitude" leitmotiv is missing in Donington. See for example Bribitzer-Stull 2015, 123.
[36] The same pair of leitmotifs starts together with a vertical major seventh. While, in Example 20 [Bach Prelude], that seventh functions in relation to the preceding passage as an upper 7–8 appoggiatura, a moment where it is approached as a modification of a strong cadence in a local

I have found very few re-interpreted dyads that are not thirds, although I cannot rule out the possibility that future research will find more varied such examples. Generalization concerning such dyads is possible based on abstract investigation. See Appendix 4. Several factors are involved: a) The identity of the diatonic interval class: fifths (and fourths) or sevenths (and seconds);[37] b) The location of the dyad members in the complete implied chord. This is more varied for fifths than for sevenths; c) The quality of the dyad. The diminished intervals hardly allow for re-interpretation; d) The quality of the potential chords that include the dyad. Even when the location of the dyad in the chord is unchanged (in Example 22: F and E as root and seventh), the chord quality may change (in Example 22: from minor-major to major-major).

key takes place in *Das Rheingold*, Scene 3, at Wotan's text "Sein' harren wir hier." See Kurth 1920/22, 456, Example 258. In *Das Rheingold*, the "Mime as smith" leitmotif still has that more general sense of forging. See Donington 1963/74, ibid.

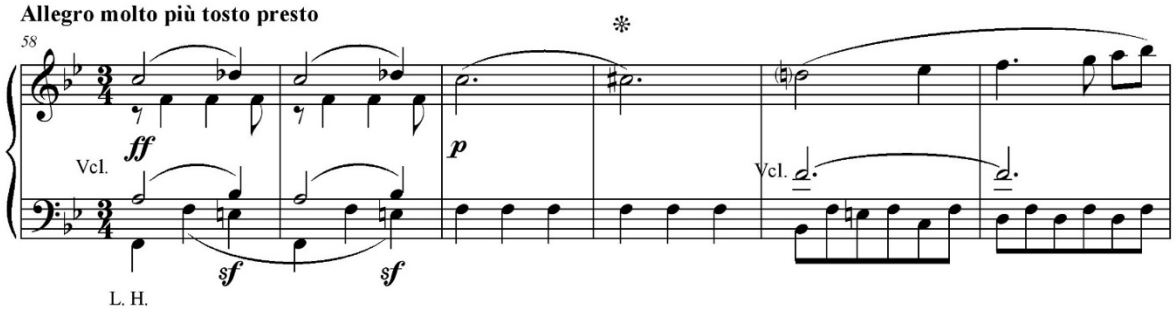
[37] For diatonic interval classes, see for example Quinn 2010.

§7 DYADS THAT UNDERGO ENHARMONIC REINTERPRETATION

So far, all the dyads we have encountered preserved their intervallic identity. Occasionally, unaccompanied intervals can nevertheless undergo enharmonic reinterpretation. It should go without saying that also in the ordinary enharmonic transformation of complete chords it is the tendency of single tones that changes (Bass 2007, Muniz 2019), modifying the identity of member dyads within the chord. Enharmonic transformation of dyads can also occur when the complete chords change.

It is seldom the case, however, that the reinterpreted dyad is not part of an explicit complete chord (or chords). A simple case happens when, in spite of the enharmonic change, the referred root is constant. See Example 23. The second theme group, in B♭ major (the diatonic III of G minor), arrives at m. 62 after a long standing on its dominant (since m. 50). That prolonged dominant uses mixture, as if the minor form of the III will arrive: the fifth of the local V (the tone C) is decorated several times by its minor upper neighbor (D♭). When, in the last measure before the arrival of B♭ major, a dyad of compound eight semitones sounds alone, it is only natural to hear the upper voice as D♭, i.e., another occurrence of the minor sixth above F. Only when the major form of B♭ arrives (with the third, D, in the melody) does one understand the upper voice in retrospect as C♯, a passing chromatic tone creating the interval of an augmented fifth. This conceptual change, from the ordinary interval to its enharmonic augmented equivalent, is more normative than the other way around, as Weber claimed concerning the first measure of Mozart's Quartet K. 465 (see Example 1).

EXAMPLE 23. Beethoven, Cello Sonata Op. 5, No. 2, ii, mm. 58–63. Enharmonic change from minor sixth to augmented fifth



The situation in Example 23 lacks the startling character typical of enharmonic reinterpretations. Example 24, from the famous introduction to Chopin's Piano Sonata No. 2, is more perplexing, and also has special compositional salience. The dyad that undergoes reinterpretation is the very first interval of the sonata. It represents "a highly unique category . . . of composition that begin[s] with an interval, unsupported with a clarifying chord or other element" (Mitchell 1962, 24 and 27 [his Example 21b]). The dyad is initially itself arpeggiated, hence even understanding its members as conceptually simultaneous is a product of conceptualization.[38]

The interval occupies eight semitones (major sixth or diminished seventh). Listening without a score, there might be an aural prejudice to conceive it according to the simpler, diatonic, meaning as a major sixth (E+C♯). A possible conflicting clue in favor of a diminished seventh (E+D♭) in real-time listening is the association to a familiar precedent: the opening of Beethoven's Piano Sonata Op. 111. The notation indeed determines an opening diminished seventh, an applied vii°7 that resolves to the V, and the global context eventually confirms the sense of a diminished seventh.[39] The immediate harmonization at m. 2, however, initially forces reading a sixth (part of

[38] We have encountered non-simultaneous dyads in Examples 2 and 14, but in these cases the melodic motion clearly served as a chordal skip within a conceptually simultaneous dyad.
[39] See a similar transformation of a major sixth ($\hat{3}$ above $\hat{5}$ in E major) into a diminished seventh ($\hat{6}$ above $\hat{7}$ in C harmonic minor) in the link between movements 2 (in E major) and 3 (in C minor)

a first-inversion minor triad). The alternative would require an enharmonic reading of the minor 6/3 chord itself as E/G♯/D♭, i.e., VII [♭]7/♯3 of V. That option could perhaps be justified, but in retrospect only.[40]

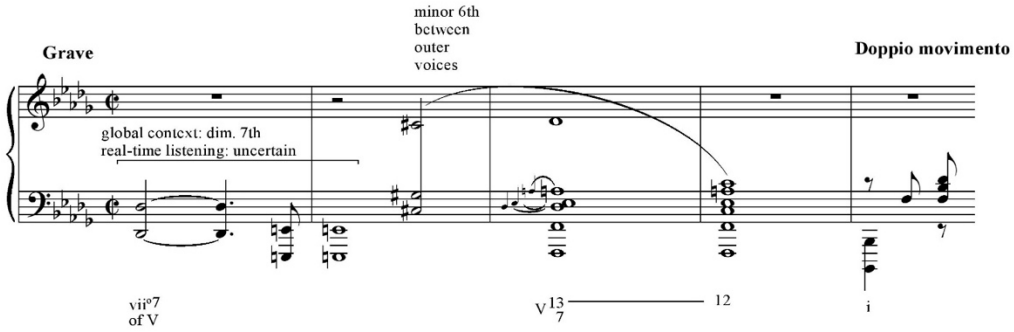
The repeat of the exposition includes, contrary to accepted tradition, the slow introduction (Rosen 1995, 179–81). The new context introduces new light on the implied harmonies (Example 24b). Coming after V13/7 of III, the opening unaccompanied D♭ serves as the bass of an implied III (D♭ major). In the new context, the C♯m 6/3 triad in the next measure is better understood as a notational enharmony of D♭m 6/3, the minor form of III. In this perspective, a notation of the introduction with F♭ in the bass of m. 2 (as Lewin [1987, 86] offers) makes sense. In the repeat, the melodic D♭–E interval is not any more a conceptually vertical dyad at all.

in Beethoven's Piano Concerto No. 3. Rings (2011, 65) brings this example in his discussion of "pivot intervals.".

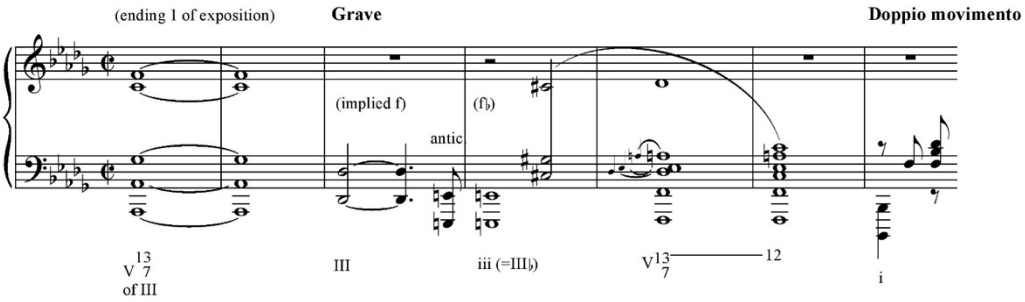
[40] Swinden (2005, 278, Example 31) suggest such a spelling, designated as D^{S}(^7)/V, a collision of dominant and subdominant elements, following Harrison 1994. For a convincing enharmonic reading of a minor 6/3 sonority as VII ♭7/♯3 see Wagner, *Das Rheingold*, Scene 4, mm. 3835–37, quoted in Cohn 2004, 311, Example 21.

EXAMPLE 24. Chopin, Piano Sonata No. 2, Op. 35, i. Complex enharmonic dyad

(a) Slow introduction (first time of mm.1–5): Opening melodic interval as arpeggiated diminished seventh, with enharmonic harmonization as a major sixth



(b) Repeat of exposition including slow introduction. Opening melodic interval does not stand for a simultaneous dyad

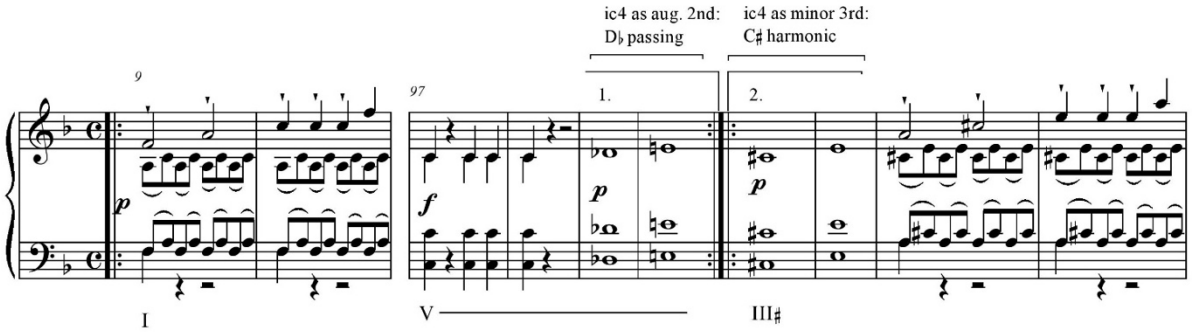


In Example 25 again a melodic interval has two meanings one of which is not a conceptually simultaneous dyad. In this case, the process goes the opposite way, transforming enharmonically a passing tone into a chord tone. In the first ending of the exposition, the harmony is V of F major, using the mixture passing tone D♭ between C to E.[41] In the second ending, respelled as C♯, this tone becomes itself a member of the same harmony with E, as part of an arpeggiated A major triad (III♯). Performers can see the change on paper immediately at the second ending, but

[41] The dynamic change on the D♭ is a conflicting element that does not support the passing character of that tone.

listeners may notice the change only in retrospect two measures later, when the first theme sounds in the new key.

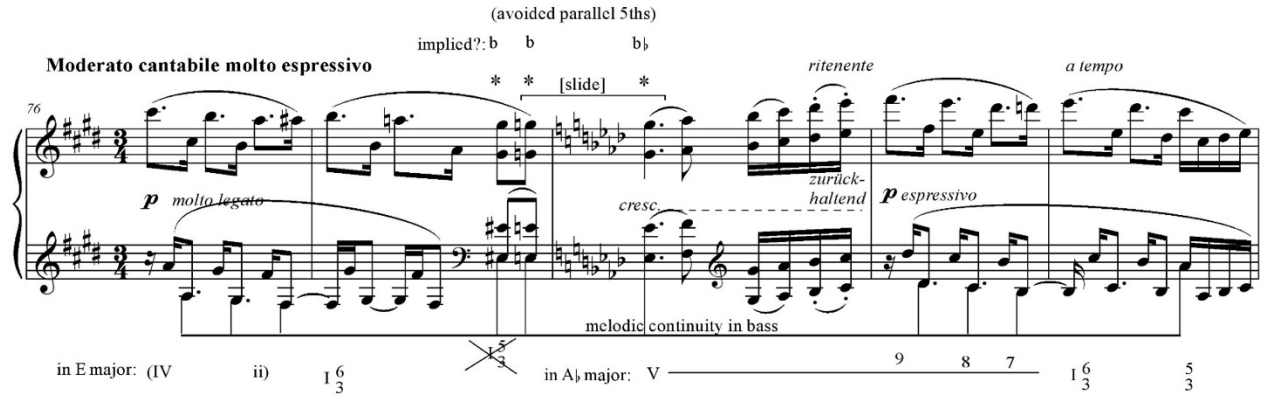
EXAMPLE 25. Haydn, String Quartet, Op. 74, No. 2, i. Enharmonic change modifies a passing tone into part of an arpreggiation.



§8 DYADS THAT DO NOT IMPLY COMPLETE CHORDS

Throughout this study, it has been taken for granted that dyads in tonal music imply complete harmonies. Our last example (Example 26) might form an exception to that norm.

EXAMPLE 26. Beethoven, Piano Sonata Op. 110, i, mm. 76–80. Bare dyads in service of tonal disorientation.



The unaccompanied tenths (compound thirds) in mm. 77–78 serve within a link between two statements of a two-measure module from the secondary theme, in the remote surface keys E major (probably F♭ major in a global context) and A♭ major. The latter statement is complete: two sets of descending thirds with passing tones in parallel tenths: $\hat{6}$–$\hat{5}$–$\hat{4}$, $\hat{5}$–$\hat{4}$–$\hat{3}$ in the upper voice, $\hat{4}$–$\hat{3}$–$\hat{2}$, $\hat{3}$–$\hat{2}$–$\hat{1}$ in the bass.[42] The statement in E major, by contrast, lacks completion. The expected G♯ in the upper part arrives with E♯ rather than E♮ in the lower part. When E♮ arrives at the next eighth note, G♮ already replaces G♯. The music goes off track, as after the barline the next dyad has E♭ in the lower part. As Kamien (1976, 221–22) observes, the E♯ in the bass of m. 77, third beat, "comes as quite a shock since we expect E♮ by analogy with bar 20–21 [the parallel moment in the exposition]. This shock indicates that the second theme has been in the 'wrong' key. The E♯ is best heard as an accented chromatic passing tone to the E on the last eighth-note of the measure. The bass succession F–E–E♭ [E♯–E♮–E♭] is similar to that in bars 66–69 . . . Thus F♭ major is approached and left in the same manner."

That last dyad (E♭+G) becomes part of V of A♭. The lack of complete harmonies in this case thus serves an effect of tonal disorientation. It is not clear whether the dyad E♯+G♯ implies a chord completion (in the event that it does, it would perhaps be a diminished triad, including B). The next dyads, E+G and E♭+G, more readily imply tones that complete the harmony (B, B♭). The emerging triads E minor and E♭ major would be in slide (=P') relations, i.e. sharing the third only.[43] Another notable feature is the contra-structural continuous descent in the bass, spanning an augmented octave from A♮ to A♭, across the thematic units.[44] These details remain under the radar

[42] The surface third-progression and its transposed version combine into a melodic fourth (compare $\hat{5}$–$\hat{4}$–$\hat{3}$, $\hat{4}$–$\hat{3}$–$\hat{2}$ as a fourth progression $\hat{5}$–$\hat{4}$–$\hat{3}$–$\hat{2}$ in Schenker [1935] 197, Fig. 72, No. 1). The internal hierarchy is subject to debate, which to my ear is not essential. Even though normally in parallel tenths there is a leading voice and following voice (Franck 2018), the vertical hierarchy in this case is unclear, and, also does not influence the perception of the dyads.

[43] The term "slide" comes from Lewin 1987, 178. The term "P'" comes from Morris 1998, 187. Capuzzo (2004, 178) has already noted their equivalence.

[44] On contra-structural melodic impulses, see Samarotto 2009.

of all existing analyses.[45] The absence of the implied tones from the literal music helps to avoid parallel fifths, but the complete triads would not support any strong functional progression. The succession E♯–E♮–E♭ might indicate merely notative enharmonicism, but the apprehension of the triple succession of forms of E requires further theorizing and is not simple in this case. Currently, Example 26 is an almost singular example that warns us not to generalize too quickly about dyads in tonal music.

§9 CONTRASTING SUGGESTIONS FOR FURTHER STUDY: WINGS OF IMAGINATION, RIGOROUS CHAINS

This study has explored systematically cases where unaccompanied dyads in common-practice tonal music give rise to more than one complete harmony. Weber drew attention to the phenomenon in the first half of the nineteenth century, and various sporadic comments in the literature are attentive to such dyads in specific examples, but the taxonomy of their manifestations is new research. The organization of the present study is based on the identity of the dyads, but the findings could also be grouped by the manner in which the dyads occur. One principal category is reinterpretation through repeat—either immediate (Example 8), large-scale (Example 3), in a sequence (Example 9), or even in retrograde (Example 12). The other principal category is actual pivots: at mid-phrase (Examples 7 and 11) or at various formal junctures (conjunct in Examples 5, 25, and probably 22; disjunct in Example 6).

In some examples, the dyads are details that are mainly significant for the theoretical investigation of how dyads work in tonal music (notably Example 20). In other examples, however, the dyads serve artistic devices that deserve attention in any analysis of the respective works

[45] In the most detailed Schenkerian reading of the passage, Beach (1987, 15) only shows the dyads as passing (and he finds as more structural the chords that I regard as local subdominants). In a detailed but hardly hierarchical reading, Riemann (1920, 427) regards the passage as moving from the tonic of E major to the dominant of A♭ major. That E-major tonic, however, never exists as a simultaneous event.

(Examples 23 and 24). Often, specific examples involve theoretical considerations that are not relevant in other examples, e.g., change of perceived hypermeter (Example 17) or questions of harmonic rhythm (Examples 4 and 13). One aspect that deserves particular attention is the hermeneutical potential of dyad re-interpretation. I provided an ad-hoc glimpse into this topic in one case (Example 6), and recently Schmalfeldt (2018) found in the tricky potential of dyads in Schumann's *Papillons* a principal device of unreliable narrative. Having once tasted this sweet kind of extra-musical observations, flying on the wings of imagination, any "technical" reading might seem to be missing something. However, it is unclear how such readings might be subject to systematic theorization. Or should they be? My *Erlkönig* suggestions fit into a specific text, and they are hardly valid without the dramatic situation. Another problem is that it may be difficult to convince readers with such interpretations.

Along with exploring the tempting route of hermeneutic interpretations of dyads, I would like also to advocate contrasting paths that require more rigorous investigations. Cognitive music scholars might find interest in this clear case of gap between sound sensation and musical perception, and propose experiments to understand better the exact listening experience of dyads in common-practice tonal music.

Within traditional music theory, it would be instructive to learn the limits under which the premise that dyads represent fuller harmonies is valid. This might require studying dyads in other repertories. In pre-tonal music, implications similar to those proposed here might come to the mind of modern listeners, but create a debate as to whether they are unjustified anachronisms. Dahlhaus ([1968] 1990, 67) indeed contended categorically that in the analysis of pre-tonal music, the concept that "sonorities of two tones are interpreted as fragments of three- or four-tone sonorities" must be dropped. Further research is still required in order to shed light on the exact circumstances that should determine the sense of implied tones in early music. What is at stake here is the very legitimacy of suggesting implied tones in two-part counterpoint, at least in

Renaissance style.[46] Dyads in twentieth-century extended tonality deserve a separate study; the validity of my observations on that repertory is uncertain as well.

Still harsher, even ethical, problems emerge in the interpretation of dyads in non-Western music, especially from cultures that do not use triadic harmony. This is of course a specific case of harmonization of non-Western music in general. When Puccini, for example, harmonizes the Japanese song "Sakurah" in *Madame Butterfly* (Act 1, rehearsal no. 75), the specific harmonies he chooses depend of course on the melodic tones, but the very enterprise of harmonization is deliberately counter-stylistic with regard to the original.

While the hermeneutic path allows more subjective readings for the sake of interpretative insights that enrich the experience of musical pieces, a cautious and rigorous inquiry of the exact conditions that give rise to justified dyad interpretation may sharpen our understanding of functional tonality in general. Between the wings of imagination and the chains of rigorous analysis, the current contribution advances our understanding of the undertheorized but widely occurring reinterpretation of dyads in common-practice tonal music.

[46] See also Dahlhaus 1963. For an exceptional view, see Mozart's comment discussed in note 18 above.

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APPENDIX 1: The dyads in the musical examples and additional cases, their size, scale degrees, tonal meanings, manner of change (under repeat or as pivot) and formal locations (at formal junctures or mid-phrase). [n.h. = non-harmonic]

Ex.	Piece	Dyad	Scale degree pair[47]	1st meaning chord members	2nd meaning Chord members	Type of meaning change	Location
related to note 2	Wagner, *Tristan und Isolde*, Act 1, scene 4: m. 15 and m. 83	M3	$\hat{1}$+$\hat{3}$	I: root + 3rd (might have been)	"III 6/4" root + 3rd (realized)	Pivot	Formal juncture local
3	Bach, Cello Suite No. 3, Bourrée No. 2, mm. 9–10	M3	$\hat{3}$+$\hat{5}$	III: root + 3rd	i: 3rd + 5th	Repeat, large scale	Formal juncture
Agawu 1 Baker 6	Beethoven, Symphony No. 5, i, mm. 1–2.	M3	$\hat{3}$+$\hat{5}$	i: 3rd + 5th	III: root + 3rd	Repeat, large scale	Formal juncture
Baker 7–11	Bach, Cello Suite No. 5, Gigue, m. 1 with upbeat	M3	$\hat{3}$+$\hat{5}$	i: 3rd + 5th	III: root + 3rd	Repeat, large scale	Formal juncture
Baker 20	Mozart, Piano Sonata K. 330, ii, upbeat to m. 20	M3	$\hat{3}$+$\hat{5}$	i: 3rd + 5th	III: root + 3rd	Repeat, medium scale	Formal juncture
Baker 21	Beethoven, Piano Sonata Op. 2,1, iii, upbeat to m. 1	M3	$\hat{3}$+$\hat{5}$	i: 3rd + 5th	III: root + 3rd	Repeat, medium scale	Formal juncture
note 7	Haydn, String Quartet, Op. 33,1, i, m. 1	M3	$\hat{3}$+$\hat{5}$	III: root + 3rd	i: 3rd + 5th	Repeat, large scale	Formal juncture
related to note 7	Brahms, Clarinet Quintet, i, m. 1	M3	$\hat{3}$+$\hat{5}$	III: root + 3rd	i: 3rd + 5th	Repeat, large scale	Formal juncture
4	Bach, Invention in B♭, m.11	M3	$\hat{4}$+$\hat{6}$	IV: root + 3rd	ii: 3rd + 5th	Pivot	Mid-phrase
note 10	Mendelssohn, *Midsummer Night's Dream*, overture, m. 395	M3	$\hat{1}$+$\hat{3}$	vi: 3rd + 5th	I: root + 3rd	Pivot	Formal juncture, conjunct
related to 4	Beethoven, Piano Sonata, Op. 10,3, iv, mm. 37 and 39	M3	$\hat{1}$+$\hat{3}$ (in ♭VI)	I: root + 3rd	vi: 3rd + 5th	Pivot	Mid-phrase
		m3	$\hat{6}$+$\hat{8}$ (in ♭VI)	vi: root + 3rd	IV: 3rd + 5th	Pivot	Mid-phrase
related to 4	Bach, English Suite No. 5, Sarabande, m. 7 beat 1	M3	$\hat{3}$+$\hat{5}$ (in Vm)	III: root + 3rd	i: 3rd + 5th	Pivot	Mid-phrase
related to 4	Schumann, Novelette Op. 21,5, m. 115	M3	modulatory context	E♭: root + 3rd (preferable)	c: 3rd + 5th	Pivot	Mid-phrase
related to 4?	Bach, English Suite No. 3, Prelude, m. 135	m3	modulatory context	d: root + 3rd	vii°/c: root + 3rd	Pivot	Mid-phrase
5	Mendelssohn, Song without Words, Op. 102,2, m. 20	m3	$\hat{3}$+$\hat{5}$	iii: root + 3rd	I: 3rd + 5th	Pivot	Formal juncture, conjunct

[47] For the sake of comparison, every raised scale degree is labeled as ♯ and any lowered scale degree as ♭.

Ex.	Piece	Dyad	Scale degree pair	1st meaning chord members	2nd meaning Chord members	Type of meaning change	Location
note 11	Schubert, Symphony No. 9, i, 591–98	M3	♭$\hat{3}$+$\hat{5}$	i [=I♭]: 3rd + 5th (as cad. 6/4)	♭III: root + 3rd	Pivot	Formal juncture, re-interpreted
related to 5	Beethoven, *Missa Solemnis*. link from Christe Eleison to Kyrie 2	m3	$\hat{3}$+$\hat{5}$	iii: root + 3rd	I: 3rd + 5th	Pivot	Formal juncture, disjunct
6	Schubert, *Erlkönig*, m. 80	m3	♯$\hat{3}$+$\hat{5}$	I [=i♯]: root + 3rd	♯iii: 3rd + 5th	Pivot	Formal juncture, disjunct
7	Schumann, *Carnaval* #21, m. 1 vs. m. 7	m6	$\hat{1}$+$\hat{3}$	I: root + 3rd	vi: 3rd + 5th (as cadential 6/4)	Pivot	Mid-phrase
8	Mozart, Piano Sonata K.309, i, m. 15 vs. m. 18	m3	$\hat{6}$+$\hat{8}$	IV: 3rd + 5th	vi: root + 3rd	Pivot and also repeat	Formal juncture
9	Bach, Sinfonia in D, mm. 1–2	M3, m3 in sequence	$\hat{4}$+$\hat{6}$, $\hat{3}$+$\hat{5}$	IV: root + 3rd, better than ii: 3rd + 5th	I: 3rd + 5th Not iii: root + 3rd	Repeat in sequence	Mid-phrase
		D5, P5 in sequence	$\hat{3}$+♭$\hat{7}$, $\hat{2}$+$\hat{6}$	V6/5 of IV: 3rd + 7th, better than vii°7/ii: 3rd + 7th	vii 6/5: 3rd + 7th		
11	Chopin, Mazurka Op. 30,2, mm. 33–40	M3	$\hat{4}$+$\hat{6}$	vi: 3rd + 5th also IV: root + 3rd	V9: 7th + 9th	Repeat	Throughout phrase [explicit harmonies]
12	Haydn, Piano Sonata Hob. XVI:26, iii, m. 2 vs. m. 19	m3	$\hat{2}$+$\hat{4}$	V7: 5th +7th	ii: root + 3rd	Repeat in retrograde	Mid-phrase
note 22	Schubert, Piano Sonata D.850, ii, m.16 beat 3	m3	$\hat{2}$+$\hat{4}$	ii: root and 3rd	V7: 5th + 7th	Alternate readings	Mid-phrase
14	Chopin, Mazurka Op. 59,1, m. 8	m3	$\hat{3}$+$\hat{5}$	III: root + 3rd	Fr/V: 5th +7th	Pivot	Mid-phrase
related to 14	Mozart, *Maurerische Trauermusik*, K.477, m. 9	M3	$\hat{1}$+$\hat{3}$	i: root + 3rd	Gr: 3rd + 5th	Pivot	Beginning of phrase
15	Haydn, Trumpet Concerto, iii, mm. 177–78	m3	$\hat{2}$+$\hat{4}$	V9/vi: 7th + 9th	V7: 5th +7th	Pivot	Formal juncture, disjunct
16	Schubert, Ländler D.366,14, m. 1	m3	$\hat{3}$+$\hat{5}$	I: 3rd + 5th unrealized	V13/7: root + 13th	--	--
17	Haydn, Symphony No. 55, m. 3 vs. m. 69	M3	$\hat{1}$+$\hat{3}$	I: root + 3rd	V: n. h. 6th+8th	Repeat, remote analogous moment	Beginning of phrase
related to 17	Haydn, String Quartet, Op. 64,5, i, m. 1 vs. m. 60	M3	$\hat{1}$+$\hat{3}$	I: root + 3rd	V/vi: n. h. 6th+8th	Repeat, remote analogous moment	Beginning of phrase

Ex.	Piece	Dyad	Scale degree pair	1st meaning chord members	2nd meaning Chord members	Type of meaning change	Location
18	Haydn, String Quartet, Op. 20,1, i, mm. 1, 4 and 110vs. m. 7	both m3 and M3	various			No change	Mid-phrase
20	Bach, *The Well-Tempered Clavier*, i, Prelude in B, m. 2	M3	$\hat{1}+\hat{3}$	I: root + 3rd with implied doubled root (tentative) or implied n.h. 7th		NA	Mid-phrase
related to 20	Bach, Organ Prelude from BWV. 539, m. 16	M3	$\hat{3}+\hat{5}$	i: 3rd + 5th (in Vm) tentative	"III 6/4" root + 3rd	NA	Mid-phrase
related to 20	Bach, English Suite No. 5, Allemande, m. 7 beat 3	m3	$\hat{4}+\hat{6}$	iv: root + 3rd with implied doubled root or implied 7th		NA	Mid-phrase
21	Chopin, Fantasy Op. 49, m. 21	P5	$\hat{2}+\hat{6}$	ii: root + 5th	V9: 5th + 9th	No change	Mid-phrase
22	Wagner, *Siegfried*, prelude to Act 1, m. 122	M7	$\hat{1}+\hat{7}$ becom $\hat{6}+\hat{5}$	i/Fm: root + 7th	VI/Am: root + 7th	Pivot	Formal juncture, conjunct
note 39	Beethoven, Piano Concerto No. 3, link of ii and iii	D7/M6	$\hat{3}+\hat{5}$ become $\hat{7}+\hat{6}$	I/EM: 3rd + 5th	viiº7/Cm: 7th + root in that order	Pivot	Formal juncture, disjunct
23	Beethoven, Cello Sonata No. 2, ii, m. 61	m6/A5	$\hat{5}+\flat\hat{3}/\sharp\hat{2}$	V: root + n.h. 6th	V: root + raised 5th	Pivot	Formal juncture, disjunct
24	Chopin, Piano Sonata 2, i. mm. 3–4	D7/M6	$\hat{3}+\hat{7}$ [harmonic]/$\flat\,\hat{8}$	viiº7/V: root + 7th	III$\flat$: root + 3rd	Pivot and also repeat	Formal juncture
25	Haydn, String Quartet, Op. 74,2, i, mm. 99–100	A2/m3	$\hat{6}+\hat{7}$ become $\sharp\hat{5}+\hat{7}$	V: p.t. + 3rd	III$\sharp$: 3rd + 5th	Pivot and also repeat	Formal juncture
26	Beethoven, Piano Sonata Op. 110, i, mm. 77–78	m3	No clear key	Not implied		NA	Formal juncture
related to 26	Schubert, *Willkommen und Abschied*, D.767, m. 32	m3	Unclear (slide progression)	aº	a	Pivot	Formal juncture (internal)
related to 26	Schubert, Piano Sonata, D.784, i, mm. 47–50	m3	$\flat\hat{2}+\natural\hat{7}$ ($\sharp\hat{1}+\times\hat{6}$?)	Not implied		NA	Formal juncture disjunct

APPENDIX 2. Diatonic thirds and their participation in diatonic triads in major and minor keys[48]

Scale degrees	**Quality of the third**	**Lower dyad of**	**Upper dyad of**
	in major and in natural [n] or harmonic [h] minor keys		
$\hat{1}+\hat{3}$	M3; m3	I; i	vi; VI
$\hat{2}+\hat{4}$	m3; m3	ii; ii^o	vii^o; VII [n] or vii^o [h]
$\hat{3}+\hat{5}$	m3; M3	iii; III	I; i
$\hat{4}+\hat{6}$	M3; m3	IV; iv	ii; ii^o
$\hat{5}+\hat{7}$	M3; m3 [n] or M3 [h]	V; v [n] or V [h]	iii; III
$\hat{6}+\hat{8}$	m3; M3	vi; VI	IV; iv
$\hat{7}+\hat{2}$	M3; M3 [n] or m3 [h]	vii^o; VII [n] or vii^o [h]	**V**; v [n] V [h]

[48] As is well known, in minor, III uses the natural form only, V and VII are brought in both natural and harmonic minor.

APPENDIX 3. All consonant thirds in major and minor keys, demonstrated in relation to C major and minor. Roman numerals do not account for chord quality, whereas major keys are upper case and minor keys are lower case. Accidentals refer to raised and lower notes from the diatonic grid. All raised and lowered tones are indicated, following Goldenberg 2018, table 2.

Members of third (quality)	**Scale degrees in C major and (natural/harmonic) minor**	**Lower dyad of [triad name; Scale degree in C major and C minor [natural/harmonic]**	**Upper dyad of [triad name; Scale degree in C major and C minor [natural/harmonic]**
C+E♭ (m)	$\hat{1}$+♭$\hat{3}$; $\hat{1}$+$\hat{3}$	c: I♭; I	A♭: ♭VI (♭5); VI
C+E (M)	$\hat{1}$+$\hat{3}$; $\hat{1}$+♯$\hat{3}$	C: I; I♯	a: VI; ♯VI (♯5)
C♯+E or D♭+F♭ (m)	♯$\hat{1}$+$\hat{3}$; ♯$\hat{1}$+♯$\hat{3}$ or ♭$\hat{2}$+♭$\hat{4}$	d♭: ♭II♭ (♭5); ♭II♭	A: VI♯; ♯VI♯ (♯5)
D♭+F (M)	♭$\hat{2}$+$\hat{4}$	D♭: ♭II (♭5); ♭II	b♭: ♭VII♭; VII♭ [n]
D+F (m)	$\hat{2}$+$\hat{4}$	d: II; **d°**: II	**b°**: VII ; **b°** or B♭: VII [h] or VII [n]
D+F♯ (M)	$\hat{2}$+♯$\hat{4}$	D: II♯ [=V/V]; II♯ (♯5)	b: VII ♯5; VII ♯5 [h]
D♯+F♯ or E♭+G♭ (m)	♯$\hat{2}$+♯$\hat{4}$ or ♭$\hat{3}$+♭$\hat{5}$	e♭: ♭III♭ (♭5); III♭	B: VII ♯5/♯3; VII ♯5/♯3 [h]
E♭+G (M)	♭$\hat{3}$+$\hat{5}$; $\hat{3}$+$\hat{5}$	E♭: ♭III (♭5); III	c: I♭; I
E+G (m)	$\hat{3}$+$\hat{5}$; ♯$\hat{3}$+ $\hat{5}$	e: III; ♯III (♯5)	C: I; I♯
F♭+A♭ or E+G♯ (M)	$\hat{3}$+♯$\hat{5}$; ♯$\hat{3}$+♯$\hat{5}$	E: III♯; ♯III♯ (♯5)	d♭: ♭II♭ (♭5); ♭II♭
F+A♭ (m)	$\hat{4}$+♭$\hat{6}$; $\hat{4}$+ $\hat{6}$	f: IV♭; IV	D♭: ♭II (♭5); **d°**: II or D♭: ♭II
F+A (M)	$\hat{4}$+ $\hat{6}$; $\hat{4}$+♯$\hat{6}$	F: IV; IV♯	d: II; II ♯5
F♯+A (m)	♯$\hat{4}$+$\hat{6}$; ♯$\hat{4}$+♯$\hat{6}$	f♯: ♯IV (♯5); ♯IV (♯5/♯3)	D: II♯; II♯ (♯5)
G♭+B♭ or F♯+A♯ (M)	♯$\hat{4}$+♯$\hat{6}$; ♯$\hat{4}$+×$\hat{6}$	G♭:♭V (♭5/♭3);♭V (♭5/♭3)[h] or F♯: ♯IV (♯5/♯3); ♯IV (♯5/×3)	e♭: ♭III♭ (♭5); III♭
G+B♭ (m)	$\hat{5}$+♭$\hat{7}$; $\hat{5}$+$\hat{7}$ [n]	g: V♭; V [n]	E♭: ♭III (♭5); III
G+B (M)	$\hat{5}$+$\hat{7}$; $\hat{5}$+$\hat{7}$ [h]	G: V; V [h]	e: III; ♯III (♯5)
A♭+C♭ or G♯+B (m)	♭$\hat{6}$+♭$\hat{8}$ or ♯$\hat{5}$+$\hat{7}$; $\hat{6}$+♭$\hat{8}$ or ♯$\hat{5}$+$\hat{7}$ [h]	a♭: ♭VI♭ (♭5); VI♭	E: III♯; ♯III♯ (♯5)
A♭+C (M)	♭$\hat{6}$+$\hat{8}$; $\hat{6}$+$\hat{8}$	A♭: ♭VI (♭5); VI	f: IV♭; IV
A+C	$\hat{6}$+ $\hat{8}$; ♯$\hat{6}$+ $\hat{8}$	a: VI; ♯VI (♯5)	F: IV; IV♯
A+C♯	$\hat{6}$+♯$\hat{8}$; ♯$\hat{6}$+♯$\hat{8}$	A: VI♯; ♯VI♯ (♯5)	f♯: ♯IV (♯5); ♯IV (♯5/♯3)
B♭+D♭ or A♯+C♯	♭$\hat{7}$+♭$\hat{2}$; $\hat{7}$ [n]+♭$\hat{2}$	b♭: ♭VII♭; VII♭ [n]	G♭: ♭V (♭5/♭3);V (♭5/♭3) [h] or F♯: ♯IV (♯5/♯3); ♯IV (♯5/×3)
B♭+D	♭$\hat{7}$+$\hat{2}$; $\hat{7}$ [n]+$\hat{2}$	B♭: ♭VII ; VII [n]	g: V♭; V [n]
B+D	$\hat{7}$+$\hat{2}$; $\hat{7}$ [h]+$\hat{2}$	**b°**: VII; VII [h]	G: V; V [h]
C♭+E♭ or B+D♯	♭$\hat{1}$+♭$\hat{3}$ or $\hat{7}$+♯$\hat{2}$; ♭$\hat{1}$+ $\hat{3}$ or $\hat{7}$ [h]+♯$\hat{2}$	B: VII ♯5/♯3; VII ♯5/♯3 [h]	a♭: ♭VI♭ (♭5); VI♭

APPENDIX 4. Possible chord types for non-tertian diatonic interval classes

	root and 5th	3rd and 7th	5th and 9th	root and 7th	3rd and 9th
perfect fifth	Major triad Minor triad Major-major 7th Dominant 7th Minor-major 7th Minor-minor 7th	Major-major 7th Minor-minor 7th Half-dim. 7th	Major V9 Ex. 20 Min-Maj-Maj [II9 in major] Maj-maj-min [I9 in major]		
diminished fifth	Dim. triad Dim. 7th Half-dim. 7th	Dim. 7th Dominant 7th	Minor V9		
minor seventh				Dominant 7th Minor-minor 7th Half-dim. 7th	
Major seventh				Major-major Minor-major [both in Ex. 21]	Major V9
Diminished seventh				Dim. 7th	Minor V9

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Yosef Goldenberg teaches at the Jerusalem Academy of Music and Dance, where he also serves as head librarian. He is a theorist of tonal music and scholar of Israeli music. He is the author of *Prolongation of Seventh Chords in Tonal Music* (Edwin Mellen Press, 2008) and co-editor of *Bach to Brahms: Essays on Musical Design and Structure* (University of Rochester Press 2015), winner of the Society for Music Theory's 2016 Outstanding Multi-Author Publication Award. On music theory, he also published in *Indiana Theory Review*, *Intégral*, *Journal of Music Theory*, *Journal of Schenkerian Studies*, *Music Analysis*, *Music Theory and Analysis*, *Music Theory Online*, and *Theory and Practice*. Currently, Yosef Goldenberg works on a full-length study of enharmonicism.